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Set Theory

Topic	Interpretation
Sets A set is a collection of objects. These objects are called *elements* of the set. Sets are represented by **A, B, C, etc**… If A is a set and x is an element of A, we write: $x \in A$. $x \notin A$ means x is not an element of A. sets may be described by a common property of its elements rather than by a list of its elements: $\{x \mid x$ has a property $P\}$ read: the set of all elements x such that x has property P. A set with no elements is called the *empty* set : ϕ.	Example1: $A = \{5,6,7\}$ $5 \in A$; $8 \notin A$ Example2: $A=\{x \mid x$ is a natural number less than $5\}$ $A = \{1,2,3,4\}$ Example3: the set passengers allowed to smoke in a non smoking flight is an empty set. 0 , ϕ ,$\{0\}$ should be distinguished: 0: represents a number. ϕ: represents a set of no elements. $\{0\}$: represents a set with one element. a singleton set.
Subsets A set **A** is a *subset* of a set **B** (Written $\mathbf{A} \subseteq \mathbf{B}$) if every element of **A** is also an element of **B**. For any set **A**: 1. $\phi \subseteq \mathbf{A}$; $\mathbf{A} \subseteq \mathbf{A}$ 2. A set of *n* distinct elements, has 2^n subsets. e.g. A set of 3 distinct elements has $2^3 = 8$ subsets	Example4: **A** is the set of all small businesses with employees less than 20;**B** is the set of all businesses. Each business with employees less than 20 is also a business, so $\mathbf{A} \subseteq \mathbf{B}$ Example5: List all subsets of $\{1,5,6\}$ There are 8 subsets: ϕ , $\{1\}$, $\{5\}$,$\{6\}$ $\{1,5\}$,$\{1,6\}$,$\{5,6\}$ $\{1,5,6\}$
Universal set The universal set in a particular discussion is the set of all objects being discussed.	Example6: A company produces only two types of items Large **L** and Small **S**; The universal set here may be denoted by $\mathbf{U} = \mathbf{L} \cup \mathbf{S}$

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Venn Diagram

Are used to illustrate relationships among sets. Figure 1.1 shows a set **A** which is subset of a set **B** (**A** is entirely in **B**); the rectangle represents the universal set **U**.

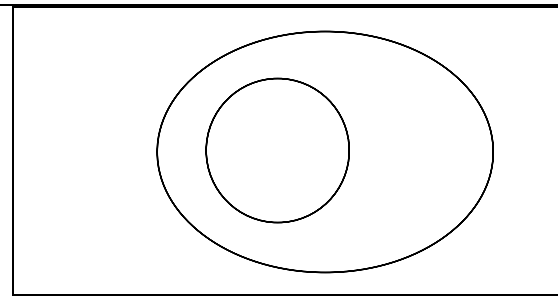


fig 1.1

Operations on sets

Let **A** and **B** be any sets with **U** the universal set then:

1. Compliment A^c:

the *compliment* of set **A** is the set of all elements of **U** which *do not* belong to **A**:

$$A^c = \{x | x \notin A \text{ and } x \in U\}$$

e.g. If **A** is the set of all female students in a class, then A^c would be the set of all male students in the class.

2. Intersection $A \cap B$:

the set of all elements belonging to *both* set **A** ***and*** set **B**:

$$A \cap B = \{ x | x \in A \text{ and } x \in B\}$$

3. Union $A \cup B$:

the set of all elements belonging to set A **or** to set B **or** to both:

$$A \cup B = \{ x \in A \text{ or } x \in B \text{ or both}\}$$

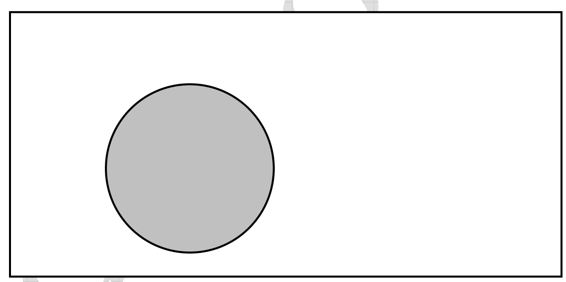
$$A \cup B = \{ 1,3,5,7,4,6\}$$

4. Related properties :

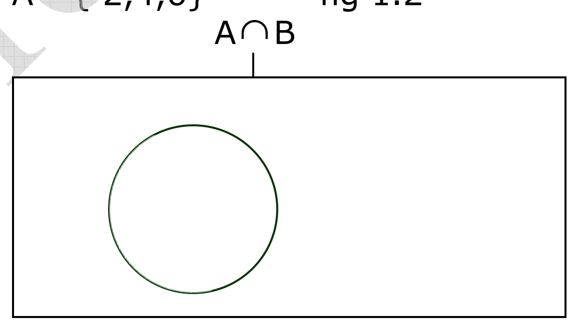
1. $A \cap A^c = \phi$
2. $A \cup A^c = U$
3. $\phi^c = U$; $U^c = \phi$
4. **Demorgan's Theorems:**
   a. $(A \cap B)^c = A^c \cup B^c$
   b. $(A \cup B)^c = A^c \cap B^c$

$U = \{1,2,3,4,5,6,7\}$; $A=\{1,3,5,7\}$

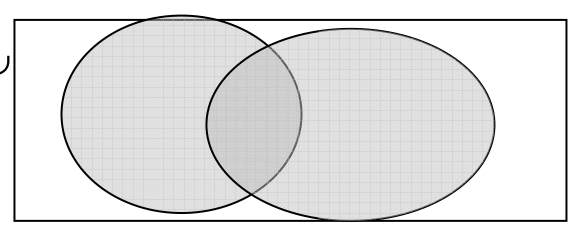
$B = \{3,4,6\}$



$A^c=\{ 2,4,6\}$ fig 1.2



$A \cap B = \{ 3 \}$ fig 1.3



A∪ 1.4

5. Order of a set

the number of elements in a set **A** is called the *order* of **A**

Written **n(A)** or |A| or n_A .

6 . Union rule for counting

$$n(A \cup B)=n(A) + n(B)- n(A \cap B)$$

Applications

1. Interpreting statements in set notation:

A good approach is to explain this by examples:

Example1:

Let **M**: the set of all students taking the management Math. course.

A: all students taking accounting.

S: All students taking statistics.

Interpret each of the following statements in set notation:

a. All students taking management math or accounting or statistics:
   $\mathbf{M} \cup \mathbf{A} \cup \mathbf{S} = \mathbf{U}$

U is the set of all students at IITM serves as Universal.

b. **T**: All students taking accounting and statistics:
   $\mathbf{T} \subseteq \mathbf{A} \cap \mathbf{S}$
c. **N**:All students not taking management math.
   $\mathbf{N} \subseteq \mathbf{M}^c$
d. **R**:All students not taking accounting and not taking statistics:
   $\mathbf{R} \subseteq \mathbf{A}^c \cap \mathbf{S}^c$

Recall that :

Union means **or**

Intersection means **and**

Compliment means **not**

Example2:

A department store classifies credit applicants by sex , marital status and employment status:

M: the set of male applicants.

S: the set of single applicants.

E: the set of employed applicants.

Describe the following sets in words:

a. $\mathbf{M} \cap \mathbf{E}$: male **and** employed
   The set of all male employed applicants.

b. $\mathbf{M}^c \cap \mathbf{S}$ **:** not male **and** single
   The set of all single female applicants.

c. $\mathbf{M}^c \cup \mathbf{S}^c$ **:** not male **or** not single
   The set of all female or married applicants.

d. $\mathbf{M} \cap \mathbf{E} = \phi$
   The set of unemployed males.

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2. Venn diagrams applications

1. Single set :

Including only a single set **A** inside the universal set, divides **U** into two nonoverlapping regions:

1: represents those elements belonging to set **A**.

2: represents $\mathbf{A^c}$, those elements outside set **A**.

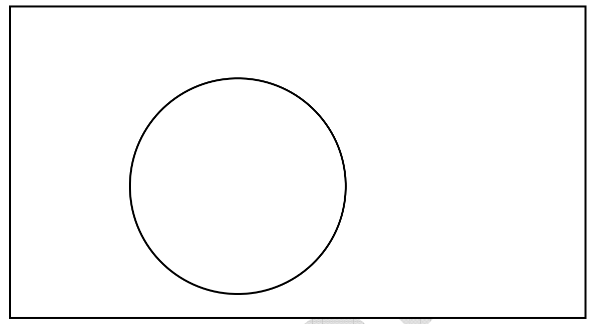

fig 1.5

2. Two sets :

Leads to 4 regions **:**

1: $\mathbf{A^c \cap B^c}$:not in **A** and not in **B**

2: $\mathbf{A \cap B^c}$: in **A** and not in **B**

3: $\mathbf{A \cap B}$: in **A** and in **B**

4: $\mathbf{A^c \cap B}$: not in **A** and in **B**

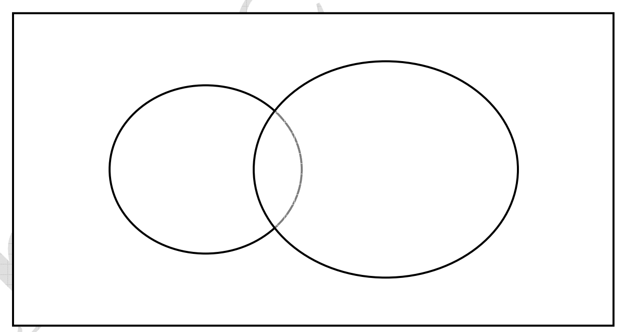

fig 1.6

3. Three sets :

Leads to 8 regions **:**

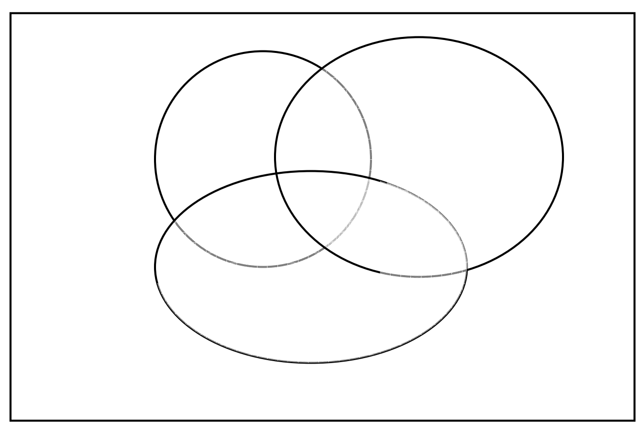

fig 1.7

<u>Example:</u> specify the region for $\mathbf{A^c \cup (B \cap C^c)}$: in **B** and not in **C** or not in **A**

Let's find $\mathbf{B \cap C^c}$ first :

B is represented by: **3,4,7,8**

$\mathbf{C^c}$ is represented by:**1,2,3,8**

The overlapping regions : **3 , 8**
Represents $\mathbf{B \cap C^c}$

$\mathbf{A^c}$ is represented by **1,6,7,8**

The union of **1,6,7,8** and **3,8** is **1,3,6,7,8** which represents $\mathbf{A^c \cup (B \cap C^c)}$

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Example:

A group of 60 students were surveyed with the following results:

19 students read Khaleej Times
18 read Gulf Today
50 read Gulf News
13 read KT and GT
11 read GT and GN
13 read KT and GN
9 read all three

a. How many students read none of the publications?

b. How many read **only** GN?

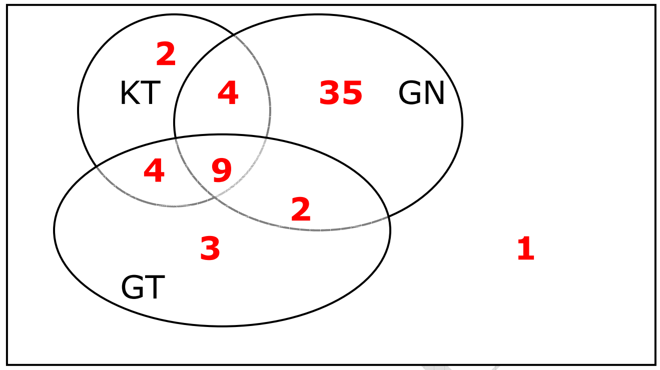
c. How many read KT and GT but not GN?

- Start by placing 9 in the area that belongs to all 3 regions.

-The region representing KT and GT is 13; 9 are allocated so the rest is 4.

- The region representing GT and GN is 11; 9 are allocated so the rest is 2.

-The region representing KT and GN is 13, so the rest is 4.



- The set KT should be 19; but 4+9+4=17 so the rest is 2.

-The set GT should be 18; but 4+9+2=15 so the rest is 3.

-The set GN should be 50; but 4+9+2=15 so the rest is 35.

-A total of: 2+4+3+2+35+4+9 = 59 are placed in various regions. Since 60 are surveyed 60 – 59 = **1 student reads none** of the publications is placed in the other regions.

- **35** read only GN.

- **4** read KT and GT but not GN.