International Institute for
Technology and Management

Tutoring Sheet # 1a

SOLUTIONS

Unit 04b : Statistics 2

1. In a group of 20 Adults , 4 out of the seven women and 2 out of 13 men wear glasses. What is the probability that a person chosen at random is a woman or wear glasses?

W: woman is chosen , P(W) = 7/20
G : person wears glasses , P(G) = 6/20
W and G **not** mutually exclusive (can happen at the same time),
$P(W \cap G) = 4/20$
$P(W \text{ or } G) = P(W) + P(G) - P(W \cap G) = 7/20 + 6/20 - 4/20 = 9/20$

2. A fair die is thrown twice. Find the probability that:

a. Neither throw result in a 4.
b. At least one throw results in a 4.

A: first throw is 4 , $P(A) = 1/6$, $P(A^c) = 1 - 1/6 = 5/6$
B: second throw is 4 , $P(B) = 1/6$, $P(B^c) = 5/6$

a. A^c and B^c are independent (either can occur without being affected by the other)
$P(A^c \cap B^c) = P(A^c) \times P(B^c) = 5/6 \times 5/6 = 25/36$

b. P(at least one throw is 4) = 1- P(Neither throw result in a 4)
= 1 – 25/36 = 11/36

3. The probability that a certain machine breaks down in the first month of operation is 0.1.If a firm has two machines installed at the same time, find the probability that at the end of the first month, just one machine has broken.

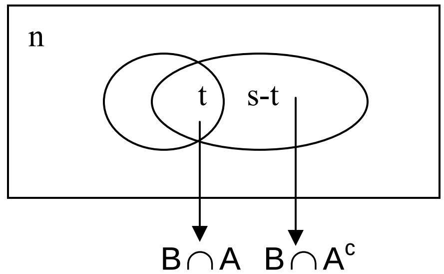
A : machine1 breaks ,$P(A) = 0.1$, $P(A^c) = 0.9$
B : machine2 breaks , $P(B) = 0.1$, $P(B^c) = 0.9$
P(just one machine breaks) : either (machine1 working and machine2 breaks) or (machine2 working and machine1 breaks)
$= P(A \cap B^c) + P(A^c \cap B) = (0.1)(0.9) + (0.9)(0.1) = 0.18$
Note that A and B^c are independent ,so are A^c and B.

4. For any two general events A and B :

a. Show that : $P(B) = P(B \cap A) + P(B \cap A^c)$

Let $n(S) = n$, $n(B) = s$, $n(A \cap B) = t \Rightarrow n(B \cap A^c) = s - t$

$$P(B) = \frac{s}{n} = \frac{t + (s - t)}{n} = \frac{t}{n} + \frac{s - t}{n}$$
$$= P(B \cap A) + P(B \cap A^c)$$



b. Deduce that : $P(B) = P(B|A) \times P(A) + P(B|A^c) \times P(A^c)$

$$P(B|A) = \frac{P(B \cap A)}{P(A)} \Rightarrow P(B \cap A) = P(B|A) \times P(A)$$

$$P(B|A^c) = \frac{P(B \cap A^c)}{P(A^c)} \Rightarrow P(B \cap A^c) = P(B|A^c) \times P(A^c)$$

$$P(B) = P(B \cap A) + P(B \cap A^c)$$
$$= P(B|A) \times P(A) + P(B|A^c) \times P(A^c)$$

c. Use part (a) to show that , If A and B are independent events, then A^c and B are independent.

We need to show : $P(B \cap A^c) = P(B) \times P(A^c)$?

$P(B) = P(B \cap A) + P(B \cap A^c)$

$P(B \cap A^c) = P(B) - P(B \cap A)$, but A and B are independent

i.e. $P(B \cap A) = P(A) \times P(B)$

$P(B \cap A^c) = P(B) - P(A) \times P(B) = P(B)[1 - P(A)]$

$P(B \cap A^c) = P(B) \times P(A^c) \Rightarrow A^c$ and B are independent.

5. A card is picked from a pack of 20 cards numbered 1,2,………,20 Given that it shows an even number, find the probability it is multiple of 4.

A: the card is even ,$A=\{2,4,6,8,10,12,14,16,18,20\}$,$n(A)= 10$

$P(A)= 10/20$

B: the card is a multiple of 4

$A \cap B = \{4,8,12,16,20\}$, $n(A \cap B)= 5$, $P(A \cap B) = 5/20$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{5/20}{10/20} = \frac{5}{10} = \frac{1}{2}$$

6. Two digits are drawn at random from a table of random numbers containing the digits 0 ,1,2,……,9. Find the probability that:

a. The sum of the two numbers is greater than 9, given that the first number is 3.

A: first number is 3 , P(A) = 1/10
B: sum of the two numbers is greater than 9
n(S)= 100 ,
$A\cap B=\{(3,7),(3,8),(3,9)\}$, $n(A\cap B) = 3$, $P(A \cap B) = 3/100$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{3/100}{1/10} = \frac{3}{10}$$

b. The second number is 2, given that the sum of the two numbers is greater than 7.

A : sum of the two numbers is greater than 7
n(A) = 64 , P(A) = 64/100
B: second number is 2
$A\cap B=\{(6,2),(7,2),(8,2),(9,2)\}, n(A\cap B)=4, P(A\cap B)= 4/100$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{4/100}{64/100} = \frac{4}{64} = \frac{1}{16}$$

c. The first number is 4, given that the difference of the two numbers is 4.

A : difference of the two numbers is 4
$A=\{(5,9),(4,8),(3,7),(2,6),(1,5),(0,4)\}$, n(A) = 6 ,
P(A) = 6/100
B: the first number is 4
$P(A\cap B)= 1/100$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{1/100}{6/100} = \frac{1}{6}$$

7. Two cards are drawn **successively** from an ordinary pack of 52 playing cards and kept out of the pack. Find the probability that:

a. Both cards are Hearts.

P(both cards are hearts) = (13/52) x (12/51)=3/51= 1/17

b. The first card is a Heart and the second card is a spade.

$P(H \cap S) = (13/52) \times (13/51) = (1/4) \times (13/51) = 13/204$

c. The second card is a diamond, given that the first card is a club.

$P(C) = 13/52$, $P(C \cap D) = (13/52) \times (13/51) = 13/204$

$$P(D|C) = \frac{P(D \cap C)}{P(C)} = \frac{13/204}{13/52} = \frac{52}{204} = \frac{13}{51}$$

8. A bag contains 4 red counters and 6 black counters. A counter is picked at random from the bag and not replaced. A second counter is then picked. Find the probability that:

a. The second counter is red, given that the first counter is red.

$P(R1) = 4/10$, $P(R1 \cap R2) = (4/10)(3/9) = 12/90$

$$P(R2|R1) = \frac{P(R2 \cap R1)}{P(R1)} = \frac{12/90}{4/10} = \frac{120}{360} = \frac{1}{3}$$

c. Both counters are red.

$P(R1 \cap R2) = (4/10)(3/9) = 12/90 = 2/15$

d. The second counter is red.

We need $P(R2 \cap R1) + P(R2 \cap B1) = 2/15 + (6/10)(4/9)$

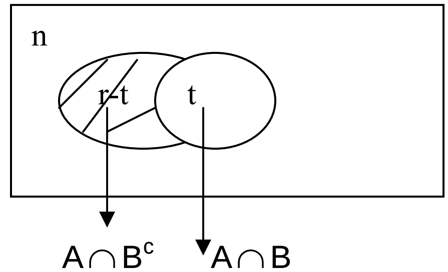
$= 2/15 + 24/90 = 2/15 + 4/15 = 6/15 = 2/5$

e. The counters are of different colors.

We need : $P(B2 \cap R1) + P(R2 \cap B1) = (4/10)(6/9)+(6/10)(4/9)$
$= 24/90 + 24/90 = 48/90$
$= 16/30 = 8/15$

9. Events A and B are such that $P(A) = 4/7$, $P(A\cap B^c) = 1/3$
$P(A|B) = 5/14$. Find

a. $P(A\cap B^c) = \dfrac{r-t}{n} = \dfrac{r}{n} - \dfrac{t}{n}$
$= P(A) + P(A\cap B)$



$P(A\cap B^c) = P(A) - P(A\cap B) \Rightarrow P(A\cap B) = P(A) - P(A\cap B^c)$
$= 4/7 - 1/3 = 5/21$

b. P(B)

$P(A|B) = \dfrac{P(A\cap B)}{P(B)} \Rightarrow P(B) = \dfrac{5/21}{5/14} = \dfrac{14}{21} = \dfrac{2}{3}$

c. $P(B|A) = \dfrac{P(B\cap A)}{P(A)} = \dfrac{5/21}{4/7} = \dfrac{7\times 5}{21\times 4} = \dfrac{5}{12}$

10. Bag A contains 5 red and 4 white counters. Bag B contains 6 red and 3 white counters. A counter is picked at random from bag A and placed in Bag B. A counter is now picked from bag B , find the probability that this counter is white.

We need (R_A & W_B) or (W_A & W_B) (Note that B will contain 10 counters when the picked one from A is put in B)
$= (5/9)(3/10) + (4/9)(4/10) = 31/90$

11. Of group of students, 56% are boys and the rest are girls. The probability that a boy is studying chemistry is 1/5 and the probability that a girl is studying chemistry is 1/11. Find the probability that a student selected at random from this group is:

a. A girl studying chemistry.

56% are boys , 44% are girls.

P(girl studying chemistry) = (44/100)(1/11)= 1/25

b. A student not studying chemistry.
P(not studying chemistry) = 1 – P(studying chemistry)

P(stud. Chem.) = P(boy stud. Chem.) + P(girl stud. Chem.)

= (56/100)(1/5) + 1/25 = 19/125

P(not studying chemistry) = 1 – 19/125 = 106/125

c. A chemistry student who is male.
P (boy | chemistry) = P(boy and chemistry) / P(chemistry)
= (14/125)/(19/125)
= 14/19

12. Three people decide to enter a marathon race. The respective probabilities that they will complete the race are 0.9,0.7 and 0.6 Find the probability that at least two will complete the race. Assume that the performance of each is independent of the performances of the others.

A : first person completes the marathon , P(A) = 0.9
B : second person completes the marathon , P(B) = 0.7
C : third person completes the marathon , P(C) = 0.6

We need :

P(at least two complete the marathon)

= P(two complete marathon) + P(three complete marathon)

P(two out of three complete marathon)

$= P(A\cap B\cap C^c) + P(A\cap B^c\cap C) + P(A^c\cap B\cap C)$
$= (0.9)(0.7)(1-0.6) + (0.9)(1-0.7)(0.6) + (1-0.9)(0.7)(0.6)$
$= 0.456$

P(three complete marathon) $= P(A\cap B\cap C)$
$= (0.9)(0.7)(0.6)$
$= 0.378$

P(at least two complete the marathon) = 0.378 + 0.456
= 0.834