04b Sample Examination Problems Chapter 7 SOLUTIONS

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1. Why do we work out a confidence interval for the difference between the means of two populations rather than comparing the separate intervals for each population mean?

Assume that we have two sample means : $\bar{x}$ **and** $\bar{y}$ **and we wish to determine whether or not** $\mu_x - \mu_y = 0$
For simplicity assume that the variances are known and equal $\sigma_x^2 = \sigma_y^2 = \sigma^2$

Let's find separate 95% C.I. for each sample mean:

The half width of each C.I. is $1.96\frac{\sigma}{\sqrt{n}}$ **the C.I. won't overlap unless** $\bar{x} - \bar{y} \succ 2\left(1.96\frac{\sigma}{\sqrt{n}}\right)$

Remember:We need the C.I. to not overlap so we can conclude that there is a real difference between the two groups.

If we have one C.I. then : $\bar{x} - \bar{y} \succ \sqrt{2}\left(1.96\frac{\sigma}{\sqrt{n}}\right)$

Since $\sqrt{2}\left(1.96\frac{\sigma}{\sqrt{n}}\right)$ **<** $2\left(1.96\frac{\sigma}{\sqrt{n}}\right)$ **then we achieve a significant difference for smaller differences between** $\bar{x}$ **and** $\bar{y}$ **,therefore using one C.I. is more powerful.**

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2. A random sample of 10 observations from a normal distribution with mean μ and variance σ^2 gives a sample mean of 1.2. An independent random sample of size 20 from the same population has sample variance 3.6. Find a 90% confidence interval for μ.

Sample1 : $n_1 = 10$, $\bar{x} = 1.2$, $s^2 = ?$

Sample1 : $n_2 = 20$, $s^2 = 3.6$, $\bar{x} = ?$

Both taken from the same Normal population N(μ, σ^2)

We need 90% C.I. for μ

<u>Remember</u> $\bar{X} \sim N(\mu, \frac{\sigma^2}{\sqrt{n}})$ and $\bar{X}$ is unbiased estimator of μ

i.e. $E(\bar{X}) = \mu$

The problem with this question is that we don't have $s^2 = ?$

For Sample1 and $\bar{x} = ?$ for sample2 so we need to estimate them.

The point estimate of 1.2 is an unbiased estimate of μ similarly We know that the sample variance s^2 is an unbiased estimator of σ^2 i.e. $E(s^2) = \sigma^2$

So we use $\bar{x} = 1.2$ and $s^2 = 3.6$

The other problem is to decide which Sample to use , As stated before the size of the sample affects the variance and therefore we would rather use Sample2 because as n increases the variance decreases.(less variability)

90% C.I. for μ : $(1-\alpha)100\% \Rightarrow \alpha = 0.1$

Here σ^2 is unknown and the sample size < 30 so we use :

$$\bar{x} \pm t_{\frac{\alpha}{2}, n-1} \frac{s}{\sqrt{n}} = \bar{x} \pm t_{0.05,19} \frac{s}{\sqrt{n}} \quad , \quad t_{0.05,19} = 1.729 \text{ (Table 10)}$$

$$= 1.2 \pm 1.729 \frac{\sqrt{3.6}}{\sqrt{20}} = 1.2 \pm 0.7336$$ **The C.I. = (0.4664 , 1.9336)**

This means : we are 90% confident that μ is somewhere between 0.4664 and 1.9336