04b Sample Examination Problems Chapter 5 SOLUTIONS

1. Write down the sample space of samples of size two without replacement from the population of three persons A, B and C.

Population size N = 3 : A ,B ,C
n = 2 without replacement

Assuming order does not matter.(Combination)

There are $C_2^3 = \frac{3!}{2!1!} = 3$ **ways**

$\Omega = \{AB, AC, BC\}$

Assuming order does matter (Permutation):

$P_2^3 = \frac{3!}{(3-2)!} = \frac{3!}{1!} = 6$ **ways**

$\Omega = \{AB, BA, AC, CA, BC, CB\}$

2. Show that the variance of the mean of a random sample of size n taken from a large population is equal to the population variance divided by the sample size.

$E(X_i) = \mu$, $Var(X_i) = \sigma^2$

$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}$ **an unbiased estimator of** μ

$$E(\bar{X}) = E\left(\frac{\sum_{i=1}^{n} X_i}{n}\right) = \frac{1}{n} E\left(\sum_{i=1}^{n} X_i\right) = \frac{1}{n}\sum_{i=1}^{n} E(X_i) = \frac{1}{n}\sum_{i=1}^{n} \mu = \frac{1}{n}(\mu + \mu + \ldots + \mu)$$

μ **is added n times** $= \frac{1}{n}(n\mu) = \mu$

$Var(\bar{X}) =$

$$Var\left(\frac{\sum_{i=1}^{n} X_i}{n}\right) = \frac{1}{n^2} Var\left(\sum_{i=1}^{n} X_i\right) = \frac{1}{n^2}\sum_{i=1}^{n} Var(X_i) = \frac{1}{n^2}\sum_{i=1}^{n} \sigma^2 = \frac{1}{n^2}(n\sigma^2) = \frac{\sigma^2}{n}$$

3. Show that the [illegible] binomial distribution with n trials and probability of success π has mean $n\pi$ and variance $n\pi(1-\pi)$.

The proof is in the "Special Distributions" page 4.
Here is an easier proof :
We'll use the fact that the binomial distribution is the sum of n independent Bernoulli trials of success probability π

X	0	1
P(X=x)	1-π	π

We first find the mean and the variance of this single Bernoulli trial and then we find the mean and the Variance of the sum of n independent Bernoulli trials Which is the same as mean and the variance of the Binomial distribution:

$$E(X) = \sum_{x=0}^{1} x p_X(x) = (0)(1-\pi) + (1)(\pi) = \pi$$

$$Var(X) = E(X^2) - E^2(X) = (0^2)(1-\pi) + (1^2)(\pi) - \pi^2$$
$$= \pi - \pi^2 = \pi(1-\pi)$$

For the binomial distribution : $X_1, X_2, \ldots, X_n$
Independent n Bernoulli trials

$$E(\sum_{i=1}^{n} X_i) = \sum_{i=1}^{n} E(X_i) = \sum_{i=1}^{n} E(X_i)$$, **each** X_i **has** $E(X) = \pi$

$$= \sum_{i=1}^{n} \pi = n\pi$$

$$Var(\sum_{i=1}^{n} X_i) = \sum_{i=1}^{n} Var(X_i) = \sum_{i=1}^{n} \pi(1-\pi) = n\pi(1-\pi)$$