04b Sample Examination Problems Chapter 12 SOLUTIONS

1. Suppose that we have 10 random observations (x_i, y_i), $i = 1, \ldots, 10$ of random variables (X, Y) that have a bivariate normal distribution.

x_i	0.33	1.01	−0.41	2.10	−0.29	−1.27	−0.48	0.31	−1.56	−0.81
y_i	1.60	1.47	−1.82	2.50	0.34	−1.37	−0.09	0.20	−1.16	−2.16

(a) Find the sample correlation coefficient between X and Y.

Here , population correlation coefficient is unknown and is estimated by the sample corr. Coeff. r_{xy} **where** $-1 \le r_{xy} \le 1$

r_{xy} = $\frac{Cov(x,y)}{S_x S_y}$ = $\frac{S_{xy}}{S_x S_y}$ **where** S_x **,** S_y **are the standard deviations**

$\sum x_i = -1.07$, $\sum y_i = -0.49$, $\sum x_i y_i = 13.3146$, n = 10

$\sum x_i^2 = 10.8203$, $\sum y_i^2 = 22.3351$, $\bar{x} = -0.107$, $\bar{y} = -0.049$

S_x = 1.0906578 , S_y = 1.57448721

$S_{xy} = \frac{1}{n-1}\sum (x_i - \bar{x})(y_i - \bar{y})$ = $S_{xy} = \frac{1}{n-1}\sum x_i y_i - n\bar{x}\bar{y}$

= $S_{xy} = \frac{1}{9}\sum 13.3146 - 10(-0.107)(-0.049)$ **= 1.4736**

r_{xy} = $\frac{S_{xy}}{S_x S_y}$ = $\frac{1.4736}{(1.0906578)(1.57448721)}$ **= 0.8581**

Strong positive correlation between x and y since it's close to 1.

(b) Test the null hypothesis that the population correlation coefficient is equal to zero against the alternative that it is greater than zero.

H_0 : $\rho = 0$

H_1 : $\rho \succ 0$

The test statistics : TS = $\dfrac{S_\beta^2}{S^2} \sim F_{\alpha,1,n-2}$ Where $S_\beta^2 = r_{xy}^2(n-1)S_y^2$

and $S^2 = \dfrac{(n-1)S_y^2(1-r_{xy}^2)}{n-2} = \dfrac{9(1.57448721^2)(1-8.5181^2)}{8} = 0.7353$

$\dfrac{S_\beta^2}{S^2} = \dfrac{(8.5181^2)(9)(1.57448721^2)}{0.7353} = 22.344$

$F_{0.05,1,8}$ = 5.32

Since the TS value falls within the rejection region , we reject H_0 and therefore $\rho \succ 0$