

04b Sample Examination Problems Chapter 11

1. (a) Derive from first principles the least squares estimator of slope for a simple linear regression.

- (b) The table below shows the population of England and Wales in millions for years in the 19th century. Test the null hypothesis that the population regression slope is 0.21.

Year	1801	1811	1821	1831	1841	1851	1861	1871
Popn.	8.89	10.16	12.00	13.90	15.91	17.93	20.07	22.71

2. (a) Show that the least squares estimators of intercept and slope are unbiased estimators of the corresponding population parameters.

- (b) The table below shows heights in cm of male children on their fourth and fifth birthdays.

Child	1	2	3	4	5
Fourth Birthday	100.0	95.1	103.3	98.2	98.8
Fifth Birthday	105.5	101.5	110.0	104.5	104.8
Child	6	7	8	9	10
Fourth Birthday	103.0	98.6	97.5	95.3	97.7
Fifth Birthday	109.0	105.5	102.5	100.4	103.6

- Find the least squares fit (i.e. intercept and slope) of a regression model for response variable height at fifth birthday and explanatory variable height at fourth birthday, and interpret your fitted line.
 - Give a 90% confidence interval for the mean height on the fifth birthday for a height on fourth birthday of 98 cm.
 - Test the null hypothesis that the population regression slope is 0.1.
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3. (a) Derive from first principles the variance of the estimator of slope for a simple linear regression.

(b) The table below shows Regional Manufacturing Capital Stock estimates in millions of pounds sterling at 1950 prices in the West Midlands and in the East Midlands.

Year	1950	1951	1952	1953	1954	1955	1956	1957	1958
West Midlands	2649	2742	2834	2918	3001	3114	3246	3385	3495
East Midlands	1748	1810	1854	1903	1944	1982	1991	2012	2028

- i. Find the least squares fit of a regression model for response variable East Midlands Capital Stock and explanatory variable West Midlands Capital Stock.
- ii. Give the analysis of variance table for this regression.
- iii. Test the null hypothesis that the population regression slope is 0.
- iv. Are the usual assumptions for inference on a regression model satisfied in this case?