International Institute for Technology and Management

November 28th ,2005

Tutoring Sheet #6

Answers

Unit 76: Management Mathematics –Differential Equations

1. Solve the following Differential Equations:

a. $y'' = xe^x$ integrating : $y' = xe^x - e^x + c_1$ (integration by parts)
integrating once more : $y = xe^x - 2e^x + c_1x + c_2$

b. $2\sqrt{x}\frac{dy}{dx} = x^2 - 1 \Rightarrow \frac{dy}{dx} = \frac{x^2-1}{2\sqrt{x}} \Rightarrow y = \int \frac{x^2-1}{2\sqrt{x}}dx = \frac{1}{2}\int (x^2-1)x^{\frac{-1}{2}}dx$

$y = \frac{1}{2}\int (x^{\frac{3}{2}} - x^{\frac{-1}{2}})dx = \frac{1}{2}(\frac{2x^{\frac{5}{2}}}{5} - 2\sqrt{x}) + c = \frac{x^{\frac{5}{2}}}{5} - \sqrt{x} + c$

c. $(2x + 3y)dx + (y - x)dy = 0$ homogeneous of degree 1:

$\Rightarrow \frac{dy}{dx} = -\frac{2x+3y}{y-x} \Rightarrow \frac{dy}{dx} = -\frac{2+3\frac{y}{x}}{\frac{y}{x}-1} = -\frac{2+3v}{v-1}$ Let $y = vx$

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$;substitute in the eq.: $v + x\frac{dv}{dx} = -\frac{2+3v}{v-1}$

Separable: $-\frac{dx}{x} = \frac{v-1}{v^2+2v+2}dv$ integrating: $-\ln|x| = \int \frac{v-1}{v^2+2v+2}dv$

$= \int \frac{v-1}{(v+1)^2+1}dv = \int \frac{v}{(v+1)^2+1}dv - \int \frac{1}{(v+1)^2+1}dv$

Let $t = v + 1$; $dt = dv$ and $v = t-1$

$= \int \frac{t-1}{t^2+1}dt - \int \frac{1}{t^2+1}dt = \int \frac{t}{t^2+1}dt - 2\int \frac{1}{t^2+1}dt = \frac{1}{2}\ln(t^2+1) - 2\tan^{-1}t + c$

$-\ln|x| = \frac{1}{2}\ln(v^2+2v+2) - 2\tan^{-1}(v+1) + c$

$-\ln|x| = \frac{1}{2}\ln(y^2/x^2 + 2y/x + 2) - 2\tan^{-1}(y/x + 1) + c$

d. $y\frac{dy}{dx}=\sqrt{y^2+1} \Rightarrow \frac{ydy}{\sqrt{y^2+1}}=dx$ integrating (let $u=y^2+1$;ydy= ½ du)

x = - $\sqrt{y^2+1}$ + c

e. x^3 dx + $(y+1)^2$ dy = 0 ;Separable : $(y+1)^2$ dy = - x^3 dx

$4(y+1)^3 + 3x^4 + C = 0$

f. $\frac{dy}{dx}=\frac{y(x+2y)}{x(2x+y)}$ Homogeneous of degree 2 ;dividing by x^2 and

letting y = vx : $v+x\frac{dv}{dx}=\frac{v+2v^2}{2+v} \Rightarrow x\frac{dv}{dx}=\frac{v^2-v}{2+v} \Rightarrow \frac{2+v}{v^2-v}dv=\frac{dx}{x}$

$\Rightarrow \int\frac{2+v}{v^2-v}dv=\int\frac{dx}{x} \Rightarrow \int\frac{2+v}{v(v-1)}dv=\int\frac{dx}{x}$ using partial fractions:

$\frac{2+v}{v(v-1)}=\frac{a}{v}+\frac{b}{v-1} \Rightarrow$ 2+v = a(v – 1) + bv

Choose v = 1 ⇒ 3 = a(0) + b ⇒ b = 3

Choose v = 0 ⇒ 2 = a(-1) + b(0) ⇒ a = -2

$\Rightarrow \int\frac{-2}{v}dv+\int\frac{3}{v-1}dv = \int\frac{dx}{x} \Rightarrow$ -2ln|v| + 3ln|v-1| = ln|x| + C

⇒ -2ln|y/x| + 3ln|y/x – 1| = ln|x| + C

2. Solve the following differential equations:

a. $\frac{dy}{dx}+2y=8x^2-2$ First order linear : P = 2 ; Q = $8x^2$ – 2

y= $e^{\int -Pdx}\left(\int Qe^{\int Pdx}\,dx+c\right)$ **= y=** $e^{\int -2dx}\left(\int (8x^2-2)e^{\int 2dx}\,dx+c\right)$

y= $e^{-2x}\left(\int (8x^2-2)e^{2x}\,dx+c\right)$; $\int (8x^2-2)e^{2x}\,dx$ By Parts :

u = $8x^2$ – 2 ⇒ du = 16x dx ; dv = e^{2x} dx ⇒ v = ½ e^{2x}

$\int udv=uv-\int vdu$; $\int (8x^2-2)e^{2x}\,dx$ = ½ $(8x^2 – 2)$ e^{2x} – ½ $\int 16xe^{2x}dx$

By Parts again: u = x ⇒ du = dx ; v= e^{2x} dx ⇒ v = ½ e^{2x}

$\int xe^{2x}dx$ = xe^{2x} - $\int e^{2x}dx$ = xe^{2x} - ½ e^{2x}

$\int (8x^2-2)e^{2x}\,dx$ = ½ $(8x^2 – 2)$ e^{2x} - 8 xe^{2x} +$4e^{2x}$

y = $e^{-2x}\left(\tfrac{1}{2}(8x^2-2)e^{2x}-8xe^{2x}+4e^{2x}+c\right)$ = ½ $(8x^2-2)$ – 8x+ 4 + ce^{-2x}

y = $4x^2$ - 8x + 3 + ce^{-2x}

b. $x\frac{dy}{dx}+3y=2x+5 \Rightarrow \frac{dy}{dx}+\frac{3}{x}y=\frac{2x+5}{x}$;First order linear:P=$\frac{3}{x}$;Q=$\frac{2x+5}{x}$

y= $e^{\int -\frac{3}{x}dx}\left(\int \frac{2x+5}{x}e^{\int \frac{3}{x}dx}\,dx+c\right)$ **=** $e^{3\ln x}\left(\int \frac{2x+5}{x}e^{3\ln x}\,dx+c\right)$

y= $x^3\left(\int \frac{2x+5}{x}\times x^3\,dx + c\right)$ **; Recall :** $e^{-3\ln x} = e^{\ln x^{-3}} = x^{-3}$

y = $x^{-3}\left(\int (2x+5)x^2\,dx + c\right)$ = $x^{-3}\left(\int (2x^3+5x^2)\,dx + c\right)$

y = x^{-3} (½ x^4 + 5/3 x^3 + c) = ½ x + 5/3 + cx^{-3}

c. $x^2\frac{dy}{dx} + xy + y = 0 \Rightarrow x^2\frac{dy}{dx} + (x+1)y = 0 \Rightarrow \frac{dy}{dx} + (\frac{x+1}{x^2})y = 0$

First order linear:P= $\frac{x+1}{x^2}$;Q= 0 ; **y=** $e^{\int\frac{x+1}{x^2}dx}\left(\int (0)e^{\int\frac{x+1}{x^2}dx}\,dx + c\right)$

y= $ce^{\int\left(-\frac{1}{x}-\frac{1}{x^2}\right)dx}$ = c $e^{-\ln x+\frac{1}{x}}$ = c($e^{\ln x^{-1}}\times e^{\frac{1}{x}}$) = (c/x) $e^{\frac{1}{x}}$

Another method: Separable $\frac{dy}{y} = -\frac{x+1}{x^2}dx$ then integrate.

d. $\frac{dy}{dx} = (x+y)^2$

Let u = x + y ⇒ y = u – x ⇒ $\frac{dy}{dx} = \frac{du}{dx} - 1$

$\frac{dy}{dx} = (x+y)^2 \Rightarrow \frac{du}{dx} - 1 = u^2 \Rightarrow \frac{du}{1+u^2} = dx \Rightarrow$ $\tan^{-1}$ u = x + c

$\tan^{-1}$ (x+y) = x + c

e. $\frac{d^2y}{dx^2}$ - $\frac{dy}{dx}$ - 6y =3x^2 + x + 2 If y = 1 and $\frac{dy}{dx}$ = 1 when x= 0

Auxiliary Equation: $r^2 - r - 6 = 0 \Rightarrow$ r = - 2 ; r = 3

y_c = $Ae^{-2x} + Be^{3x}$

y_p = C +Dx+Ex^2 to substitute this in the equation, we need y_p' and y_p''

y_p' =D+2Ex ;y_p'' = 2E , Substituting in the equation:

2E – D – 2Ex - 6(C +Dx+Ex^2) = 3x^2 + x + 2

-6Ex^2 +(-2E -6D)x + 2E – D - 6C = 3x^2 + x + 2

-6E = 3 ⇒ E=- ½; -2E -6D=1⇒ D = 0;2E –D -6C=2 ;C=-½

y_p = - ½ x^2 – ½ ; General solution : **y = y_c + y_p**

y = $Ae^{-2x} + Be^{3x}$ - ½ x^2 – ½

y(0) = 1 ⇒ $Ae^0 + Be^0$ - ½ (0) – ½ = 1 ⇒ A + B = 3/2

y′ = $-2Ae^{-2x} + 3Be^{3x}$ - x

y′(0) = 1⇒ $-2Ae^0 + 3Be^0$ - 0 = 1 ⇒ -2A + 3B = 1

Solving simultaneously for A & B : A = 7/10 ; B = 4/5

y = $\frac{7}{10}e^{-2x} + \frac{4}{5}e^{3x}$ - ½ x^2 – ½

f. $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = e^{2x}$

Auxiliary Equation: $r^2 - r - 2 = 0 \Rightarrow r = -1 ; r = 2$

$y_c = Ae^{-x} + Be^{2x}$

Note: e^{2x} is part of y_c

The particular solution ~~$y_p = Ce^{2x}$~~ will not work!

$y_p = Ce^{2x}$ to substitute this in the equation, we need y_p' and y_p''

$y_p' = 2Ce^{2x}$; $y_p'' = 4Ce^{2x}$, Substituting in the equation:

$4Ce^{2x} - 2Ce^{2x} - 2Ce^{2x} = e^{2x} \Rightarrow 0e^{2x} = e^{2x}$??

To fix it we attach x to Ce^{2x}:

Let $y_p = Cxe^{2x}$; $y_p' = (2Cx+C)e^{2x}$; $y_p'' = (4Cx+4C)e^{2x}$

$(4Cx+4C)e^{2x} - (2Cx+C)e^{2x} - 2Cxe^{2x} = e^{2x}$

$3Ce^{2x} = e^{2x} \Rightarrow C = 1/3 \Rightarrow y_p = 1/3\, xe^{2x}$;

$y = Ae^{-x} + Be^{2x} + (1/3)xe^{2x}$

Example on the normal case: $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = e^{2x}$

Auxiliary Equation: $r^2 - 6r + 9 = 0 \Rightarrow r = 3 ; r = 3$

$y_c = (A + Bx)e^{3x}$

$y_p = Ce^{2x}$ to substitute this in the equation, we need y_p' and y_p''

$y_p' = 2Ce^{2x}$; $y_p'' = 4Ce^{2x}$, Substituting in the equation:

$4Ce^{2x} - 12Ce^{2x} + 9Ce^{2x} = e^{2x} \Rightarrow Ce^{2x} = e^{2x} \Rightarrow C = 1 \Rightarrow y_p = e^{2x}$

$y = (A + Bx)e^{3x} + e^{2x}$

g. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = \sin 2x$

Auxiliary Equation: $r^2 - 2r - 1 = 0 \Rightarrow r = 1+\sqrt{2}$; $r = 1-\sqrt{2}$

$y_c = Ae^{(1-\sqrt{2})x} + Be^{(1+\sqrt{2})x}$

$y_p = C\cos 2x + D\sin 2x$; $y_p' = -2C\sin 2x + 2D\cos 2x$

$y_p'' = -4C\cos 2x - 4D\sin 2x$, Substituting in the equation:

$-4C\cos 2x - 4D\sin 2x + 4C\sin 2x - 4D\cos 2x - C\cos 2x - D\sin 2x = \sin 2x$

$(-4D + 4C - D)\sin 2x + (-4C - 4D - C)\cos 2x = \sin 2x$

$4C - 5D = 1$; $5C + 4D = 0$ Solving simultaneously for C & D:

$C = 4/41$; $D = -5/41$; $y_p = 4/41 \cos 2x - 5/41 \sin 2x$

$y = Ae^{(1-\sqrt{2})x} + Be^{(1+\sqrt{2})x} + 4/41 \cos 2x - 5/41 \sin 2x$

h. $\frac{d^2y}{dx^2} + y = \sin x + \cos x$

Auxiliary Equation: $r^2 + 1 = 0 \Rightarrow r = \pm i$

$y_c = e^{\frac{-a}{2}x}(A\cos\alpha\, x + B\sin\alpha\, x)$; $\alpha = \frac{\sqrt{4b - a^2}}{2} = \frac{\sqrt{4(1) - 0^2}}{2} = 1$

$y_c = e^{\frac{-0}{2}x}(A\cos x + B\sin x)$; $y_c = A\cos x + B\sin x$

~~y_p = Csinx + Dcosx~~ **will not work since it's part of y_c**

Let y_p = x(Csinx + Dcosx) ; y_p' = Csinx + Dcosx +x(C cosx-Dsinx)

y_p'' = C cosx – Dsinx + C cosx – Dsinx + x(-Csinx – Dcosx)

Substituting in the equation:

2C cos x – 2Dsinx – Cxsinx –Dxcosx + Cxsinx + Dxcosx = sinx + cosx

2C = 1 $\Rightarrow$ C = ½ ; -2D = 1 $\Rightarrow$ D = - ½ $\Rightarrow$ y_p = ½ x(sinx – cosx)

y = $A\cos x + B\sin x$ + ½ x(sinx – cosx)

i. $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = e^{2x}\cos x$

Auxiliary Equation: $r^2 + r - 6 = 0 \Rightarrow$ r = 2 ; r = -3

$y_c = Ae^{-3x} + Be^{2x}$

$y_p = e^{2x}$(Csinx + Dcosx)

$y_p' = 2e^{2x}$(Csinx+Dcosx)+e^{2x}(Ccosx–Dsinx)

$= e^{2x}$[(2C-D)sinx+(C+2D)cosx];

$y_p'' = 2e^{2x}$[(2C-D)sinx+(C+2D)cosx]+e^{2x}[(2C-D)cosx–(C+2D)sinx]

$= e^{2x}$ [(3C-4D)sinx + (4C+3D)cosx]

substituting in the equation:

e^{2x} [(3C-4D)sinx + (4C+3D)cosx]+ e^{2x}[(2C-D)sinx+(C+2D)cosx]

-6 e^{2x}(Csinx + Dcosx) = e^{2x} cosx

e^{2x} [(-C -5D)sinx + (5C – D)cosx] = e^{2x} cosx

C+ 5D = 0 ; 5C – D = 1$\Rightarrow$ C = 5/26 ; D = -1/26

$y_p = e^{2x}$(5/26 sinx -1/26 cosx)

y = $Ae^{-3x} + Be^{2x}$ + e^{2x}(5/26 sinx -1/26 cosx)

j. $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 8(x^2 + \sin 2x)$

Auxiliary Equation: $r^2 - 4r + 4 = 0 \Rightarrow$ r = -2 ; r = -2

$y_c = (A + Bx)e^{-2x}$

y_p = C + Dt + Et^2 + Fsin2t + Gcos2t;differentiating and substituting in the equation: C=3,D=4,E=2,F=0,G=1

y= $(A + Bx)e^{-2x}$ + 2x^2+4x+3 + cos2x

k. $\frac{d^2y}{dx^2} + 4y = \cos 2x + \cos 4x$

Auxiliary Equation: $r^2 + 4 = 0 \Rightarrow$ r = $\pm 2i$

$y_c = e^{\frac{-a}{2}x}(A\cos\alpha x + B\sin\alpha x)$; $\alpha = \frac{\sqrt{4b - a^2}}{2} = \frac{\sqrt{4(4) - 0^2}}{2} = 2$

y_c = $e^{\frac{-0}{2}x}(A\cos 2x + B\sin 2x)$; y_c = $A\cos 2x + B\sin 2x$

y_p = **x**(Csin2x + Dcos2x)+ Esin4x + Fcos4x **since sin2x is part of y_c**

then differentiate and substitute in the equation:

y = $A\cos 2x + B\sin 2x + \frac{1}{4}x\sin 2x - \frac{1}{12}\cos 4x$

LSE Previous Papers

3. Suppose the consumer demand for a company's only product line depends upon the price according to the following formula:

   q = 50 – 40 p -7 $\frac{dp}{dt}$ + $\frac{d^2p}{dt^2}$

   and the supply function q = - 10 + 20p

i) Determine the equilibrium price and quantity if p = 5

   and $\frac{dp}{dt}$ = 31 when t = 0

   Supply = Demand

   50 – 40 p -7 $\frac{dp}{dt}$ + $\frac{d^2p}{dt^2}$ = -10 + 20p

   $\frac{d^2p}{dt^2}$ -7 $\frac{dp}{dt}$ - 60p = -60

   Auxiliary Equation: $r^2 - 7r - 60 = 0 \Rightarrow$ r = 12 ; r = -5

   p_c = $Ae^{-5t} + Be^{12t}$

   p_p = C + Dt ; y_p' = D and y_p'' = 0

   substitute in the equation: -7D – 60(C+Dt) = - 60

   -60Dt -60C -7D = -60 ; D = 0 ; C = 1

   p = $Ae^{-5t} + Be^{12t}$ + 1

   p(0) = 5 ⇒ A+B + 1 = 5 ⇒ A+B = 4

   p'= $-5Ae^{-5t} + 12Be^{12t}$; p'(0) = 31⇒ -5A + 12B = 31

   Solving simultaneously for A and B : A = 1 ; B = 3

   p= $e^{-5t} + 3e^{12t}$ + 1

ii) Produce a sketch graph of p against t and describe the behavior of p .

   1.) p'= $-5e^{-5t} + 36e^{12t} = 0$ ⇒ $-5e^{-5t}(1 - 36/5\ e^{17t}) = 0$

   ⇒ $e^{17t} = 5/36$ ⇒17t = ln(5/36) ⇒ t = - 0.11

   p = $e^{-5(-0.11)} + 3e^{12(-0.11)}$ + 1 = 1.78 + 3(0.24) + 1 = 3.52

   Vertex V (- 0.11 , 3.52)

   2.) t→-∞ , p = +∞ + 0 + 1 = +∞

   t→+∞ p = = 0 + ∞ + 1 = +∞ Directional asymptote // y-axis.

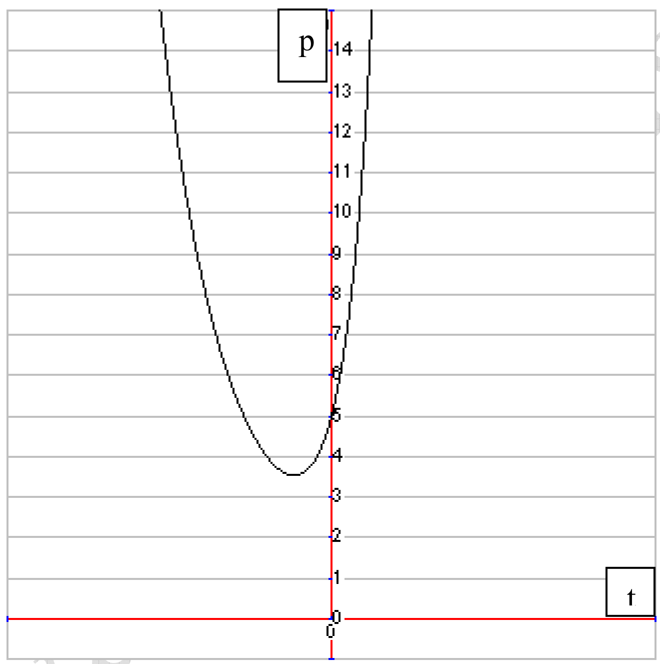
3.) Intercepts : $t = 0 \Rightarrow p = 1$; (0 ,5)

4.)

t	$-\infty$		-0.11		$+\infty$
p′		-	0	+	
p	$+\infty$	↘	3.52	↗	$+\infty$

5.) Graph :

You will lose 1 mark if You don't Label the Axes.



There is no "long run" equilibrium since p tends to infinity as t grows large.p commences on 5 and grows exponentially.

iii) Suggest how the above model might be used in practice
Do you foresee any limitations on its use. (LSE 2003)
As one of the auxiliary roots is positive,the time path diverges and hence there will be no long run stability.

4. You are given the following differential equation in y :

$$\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 12y = x\,e^{3x}$$

If $y = 1$ and $\frac{dy}{dx} = 3$ when $x = 0$

Solve the above differential equation of y , graph the solution and describe the graph in words.

(LSE 2004)

Auxiliary Equation: $r^2 - 7r + 12 = 0 \Rightarrow$ r = 3 ; r = 4

$y_c = Ae^{3x} + Be^{4x}$

y_p = ~~$e^{3x}(C+Dx)$~~ **will not work since e^{3x} is part of y_c**

To fix it we attach x to $e^{3x}(C+Dx) \Rightarrow y_p = e^{3x}(Cx+Dx^2)$

$y_p' = 3e^{3x}(Cx+Dx^2)+e^{3x}(C+2Dx)=e^{3x}[3Dx^2+(3C+2D)x+C]$

$y_p'' = 3e^{3x}[3Dx^2+(3C+2D)x+C]+e^{3x}[6Dx+3C+2D]$

$= e^{3x}[9Dx^2+(9C+12D)x + 6C + 2D]$

Substituting in the equation:

$e^{3x}[9Dx^2+(9C+12D)x+6C+2D]-7e^{3x}[3Dx^2+(3C+2D)x+C]$
$+12\, e^{3x}(Cx+Dx^2) = xe^{3x}$

$(-2Dx + 2D - C)e^{3x} = xe^{3x}$:

$-2D = 1$; $D = -\frac{1}{2}$; $C = -1 \Rightarrow y_p = e^{3x}(-x - \frac{1}{2}x^2)$

$y = Ae^{3x} + Be^{4x} + e^{3x}(-x - \frac{1}{2}x^2)$

$y(0) = 1 \Rightarrow A + B = 1$

$y' = 3Ae^{3x} + 4Be^{4x} + e^{3x}(-3/2\, x^2 - 4x - 1)$

$y'(0) = 3 \Rightarrow 3A + 4B - 1 = 3 \Rightarrow 3A + 4B = 4$

solving simultaneously for A and B : A = 0 ; B = 1

$y = Ae^{3x} + Be^{4x} + e^{3x}(-x - \frac{1}{2}x^2) = e^{4x} + e^{3x}(-x - \frac{1}{2}x^2)$

both auxiliary roots are positive ,Curve diverges
it grows exponentially as x increases.

Graph:

1.) $y' = 3Ae^{3x} + 4Be^{4x} + e^{3x}(-3/2\, x^2 - 4x - 1)$
$= 4e^{4x} + e^{3x}(-3/2\, x^2 - 4x - 1) = e^{3x}(4e^x - 3/2\, x^2 - 4x - 1)$
Apparently no vertex.

2.) $x \to -\infty$, $y = e^{4x} + e^{3x}(-x - \frac{1}{2}x^2) = e^{3x}(e^x - \frac{1}{2}x^2 - x)$;
$y \dashrightarrow (0)(0 - \infty) \dashrightarrow 0 \times -\infty$??

$e^{3x}(e^x - \frac{1}{2}x^2 - x) = \frac{e^x - 1/2\, x^2 - x}{e^{-3x}} \dashrightarrow \frac{e^x - x - 1}{-3e^{-3x}} \dashrightarrow \frac{e^x - 1}{9e^{-3x}}$

$\to \frac{0-1}{\infty} = 0$ Using Lopital's rule

Hence $y \dashrightarrow 0$; y = 0 (x-axis) is horizontal asymptote

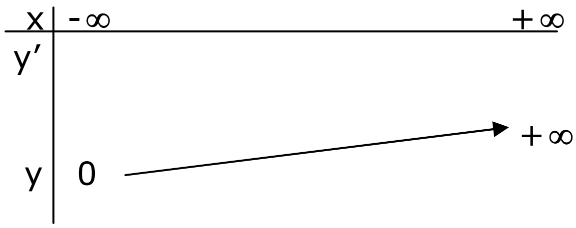
$x \to +\infty$, $y = e^{3x}(e^x - \frac{1}{2}x^2 - x) \dashrightarrow +\infty(+\infty - \infty)$??

$\frac{e^x - 1/2\, x^2 - x}{e^{-3x}} \dashrightarrow \frac{e^x - x - 1}{-3e^{-3x}} \dashrightarrow \frac{e^x - 1}{9e^{-3x}} \dashrightarrow \frac{+\infty - 1}{0} = +\infty$

Directional asymptote // y-axis

3.) Intercepts : x = 0 ; y = 1 ; (0 , 1)

4.)



5.) Graph:

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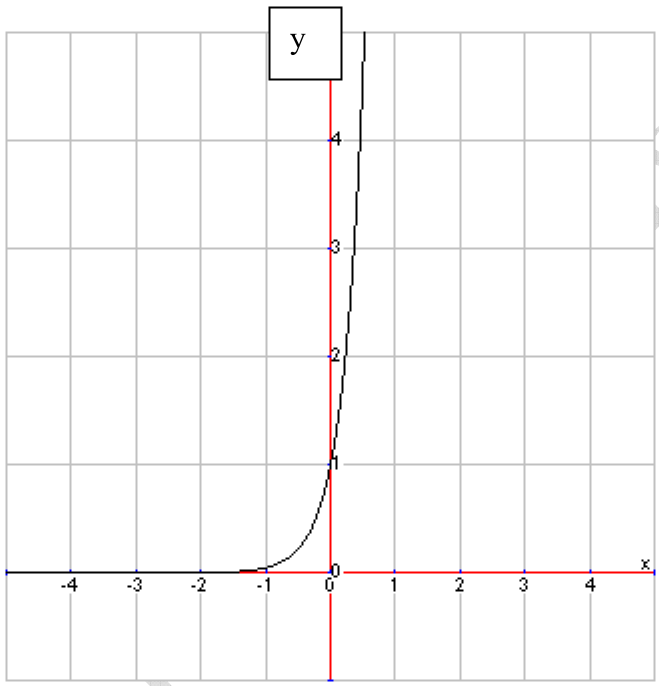

5. A company maintains its machines every t days and discovers that the overall maintenance costs of the machines, C , are related to t by the following differential equation: (LSE 2005)

$$t^2 \frac{dC}{dt} - (b-1)tC = -ab$$

where a and b are constants and $C = C_0$ when $t = t_0$

i) Derive C as a function of t and the other given constants.

$\frac{dC}{dt} - \frac{b-1}{t}C = \frac{-ab}{t^2}$ First order linear ,with $P = \frac{b-1}{t}$; $Q = \frac{-ab}{t^2}$

$$C = e^{\int -P dt}\left(\int Q e^{\int P dt}\, dt + k\right) = e^{\int \frac{b-1}{t}dt}\left(\int -\frac{ab}{t^2} e^{\int -\frac{b-1}{t}dt}\, dt + k\right)$$

$$C = e^{(b-1)\ln t}\left(\int -\frac{ab}{t^2} e^{-(b-1)\ln t}\, dt + k\right) = e^{\ln t^{b-1}}\left(\int -\frac{ab}{t^2} e^{\ln t^{1-b}}\, dt + k\right)$$

$$C = t^{b-1}\left(-ab\int \frac{t^{1-b}}{t^2}\,dt + k\right) = t^{b-1}\left(-ab\int t^{-b-1}\,dt + k\right) = t^{b-1}\left(\frac{ab}{b}t^{-b}+k\right)$$

$$C = kt^{b-1} + \frac{a}{t}$$

$$C(t_0) = C_0 \Rightarrow C_0 = k\,t_0^{b-1} + \frac{a}{t_0} \Rightarrow k = \frac{C_0 t_0 - a}{t_0^b}$$

$$C = \left(\frac{C_0 t_0 - a}{t_0^b}\right)t^{b-1} + \frac{a}{t}$$

ii) Graph C against t for the case $a = 4$, $b = 2$, $C_0 = 10$ and $t_0 = 1$

$C = 6t + \frac{4}{t}$ defined and continuous for $t \neq 0$

$C' = 6 - \frac{4}{t^2} = 0 \Rightarrow t = \pm\sqrt{\frac{2}{3}}$; 2 vertices

t--→ 0 ; C--→ ∞ ; t = 0 is a vertical asymptote

t--→ ±∞ , C --→ ± ∞ ; Directional asymptote // y-axis

Particular point : t = 1 ⇒ C = 10

t	$-\infty$		$-\sqrt{\frac{2}{3}}$		0		$\sqrt{\frac{2}{3}}$		$+\infty$
C′		+		-	‖	-		+	
C	$-\infty$	↗	y_{v1}	↘ $-\infty$	‖	$+\infty$ ↘	y_{v2}	↗	$+\infty$

You will lose 1 mark if You don't Label the Axes.