International Institute for Technology and Management

Tutoring Sheet #2 - Solution

Unit 76 : Management Mathematics

1. a. The following table shows the prices per unit of three commodities in 1995 and 2000 and the total value of purchases in those years :

Commodity	Purchases		Prices	
	1995	2000	1995	2000
X	9	15	24	20
Y	16	40	16	20
Z	18	13.5	10	15

Calculate (a) Laspeyre's and (b) Paasche's indices for the prices.

Laspeyre's $= \dfrac{\sum_i^N p_{it} \times q_{i0}}{\sum_i^N p_{i0} \times q_{i0}} \times 100 = \dfrac{770}{652} \times 100 = 118.01$

$\sum_i^N p_{it} \times q_{i0} = (20)(9)+(20)(16)+(15)(18) = 770$

$\sum_i^N p_{i0} \times q_{i0} = (24)(9)+(16)(16)+(10)(18) = 652$

Paasche's $\dfrac{\sum_i^N p_{it} \times q_{it}}{\sum_i^N p_{i0} \times q_{it}} \times 100 = \dfrac{1302.5}{1135} \times 100 = 114.75$

$\sum_i^N p_{it} \times q_{it}$ = (20 x 15) + (40 x 20) + (13.5 x 15)= 1302.5

$\sum_i^N p_{i0} \times q_{it}$ =(24x15)+ (16 x 40) + (10 x 13.5) = 1135

b. Briefly discuss the differences between Relative and Aggregate indices:

Simple Aggregate Index: This is used for a fixed group of commodities, where the quantities remain the same throughout the analysis.

Relative Aggregate Index: commodities have equal importance, that is the Index is independent of the quantities.

2. a. Briefly discuss the difficulties in creating a suitable inflation type index to determine the changing cost of living for a population of a country:

Deciding upon one index to represent the cost of living for the whole Population of the country is limited by many assumptions,approximations and Subjective data for the following reasons:

- Collection of accurate data in reasonable time.
- Data subjectivity: includes what commodities to consider and what weights to use.
- Data approximations: figures including erroneous data such as 2.5 washing Machines, 1.6 cars, etc.…
- Data updates : How often should the commodities and the weights be updated
- Index related problems:
  i. Methodology : Fixed , chained ,Paschee or Laspeyre.
  ii.Calculation : Is it possible to be calculated and how often.

b. A company wishes to know if sales in real terms have increased in the five years period 2000 – 2005.They would like to know if stock levels of their items were justified by the sales figures. Total sales for 2000 and 2005 were $ 600 000 and $ 1 200 000 respectively.

	Stock Holdings					
Year	**2000**			**2005**		
Items	**Number**	**Value$**	**Unit Price p_0**	**Number**	**Value$**	**Unit Price p_1**
A	**200**	**20 000**	**100**	**150**	**30 000**	**200**
B	**400**	**40 000**	**100**	**450**	**90 000**	**200**
C	**70**	**21 000**	**300**	**100**	**45 000**	**450**
D	**30**	**15 000**	**500**	**30**	**30 000**	**1000**

(a.) Construct a weighted index of the price increases, 2005 against 2000, for the four items of stock together.

$$\textbf{Laspeyre's} = \frac{\sum_i^N p_{it} \times q_{i0}}{\sum_i^N p_{i0} \times q_{i0}} \times 100 = \frac{181500}{96000} \times 100 = \mathbf{189}$$

$$\sum_i^N p_{it} \times q_{i0} = (200)(200)+(400)(200)+(70)(450) + (30)(1000) = \mathbf{181500}$$

$$\sum_i^N p_{i0} \times q_{i0} = (200)(100)+(400)(100)+(70)(300) +(30)(500) = \mathbf{96\ 000}$$

(b.) Calculate using the above index the percentage change of sales in real terms.

Total sales for 2000 and 2005 were $ 600 000 and $ 1 200 000 respectively.

Total sales for 2000 and 2005 = $ 600 000

2000 Sales adjusted (for price increase between 2000 & 2005)

$= \text{Total sales(in 2000)} \times \frac{Index}{100} =$ **600 000 x 189/100 = 1 134 000**

Real increase in sales = $ 1 200 000 - $ 1 134 000 = $ 66 000

3. Every month a company purchases four items in the typical Quantities and at the prices shown in the following table:

Item	Unit	Weight	Price per Unit		
			June	July	August
A	Kg	240	45	46	48
B	Kg	100	60	61	62
C	Liters	120	80	70	66
D	Thousands	200	120	130	140
			305	307	316

Using June as a base, find for July and August:

a.) The simple aggregate price index:

For July: $\frac{\sum_i^N p_{it}}{\sum_i^N p_{i0}} \times 100 = \frac{307}{305} \times 100 =$ **100.656**

For August: $\frac{\sum_i^N p_{it}}{\sum_i^N p_{i0}} \times 100 = \frac{316}{305} \times 100 =$ **103.607**

b.) The weighted aggregate price index.

Item	weight	Prices per unit					
	w_i	June p_0	$w_i p_0$	July p_1	$w_i p_1$	August p_2	$w_i p_2$
A	240	45	10800	46	11040	48	11520
B	100	60	6000	61	6100	62	6200
C	120	80	9600	70	8400	66	7920
D	200	120	24000	130	26000	140	28000
		$\sum w_i p_0 =$	**50400**	$\sum w_i p_1 =$	**51540**	$\sum w_i p_2 =$	**53640**

For July: $\dfrac{\sum_{i}^{N} p_{it} \times q_{it}}{\sum_{i}^{N} p_{i0} \times q_{i0}} \times 100 = \dfrac{\sum_{i}^{N} w_i \times p_1}{\sum_{i}^{N} w_i \times p_0} \times 100 =$ **102.261**

$$\frac{(240)(46)+(100)(61)+(120)(70)+(200)(130)}{(240)(45)+(100)(60)+(120)(80)+(200)(120)} \times 100 = \frac{51540}{50400} \times 100$$

For August: $\dfrac{\sum_{i}^{N} p_{it} \times q_{it}}{\sum_{i}^{N} p_{i0} \times q_{i0}} \times 100 = \dfrac{\sum_{i}^{N} w_i \times p_2}{\sum_{i}^{N} w_i \times p_0} \times 100 =$ **106.428**

$$\frac{(240)(48)+(100)(62)+(120)(66)+(200)(140)}{(240)(45)+(100)(60)+(120)(80)+(200)(120)} \times 100 = \frac{53640}{50400} \times 100$$

c.) If in September of the same year commodities A and B are expected to increase by 1% per kg and the price of commodity D to increase by 10% per thousand. How much must the cost per Liter of C decrease in Order that the weighted aggregate price index for September remains the same as for August?

Prices of A and B in September : (increase by 1% =1/100)
Price of A = 48 + (0.01)(48) = 48.48
Price of B = 62 + (0.01)(62) = 62.62
Price of D in September : (increase by 10% =10/100)
Price of D = 140 + (0.1)(140) = 154
Price of C decrease by x % :

Price of C = 66 – $66 \times \dfrac{x}{100}$ = 66 – 0.66x

Index remains same : $\sum w_i p_2$ = 53 640 remains same (index is not affected by $w_i p_0$) :
53640=(240)(48.48)+100(62.62)+120(66-0.66x)+(200)(154)
53640 = 11635.2 + 6262 + 7920 – 79.2x + 30800
79.2 x = 2977.2 ; x = 37.590
A price decrease of 37.6% for C in September.

4.(a) Briefly discuss the advantages and disadvantages of using Laspeyre's as opposed to Paasche's type price indices.

(b) The following table gives a index of agricultural production in the country of Groland using two index methods (The "Foodpro" index based in 1970 and the "FarmFood" index based in 2000). Also given is an estimate of population of Groland.

Year	"Foodpro" index (Base 1970=100)	"FarmFood" index (Base 2000=100)	Groland's Estimated Population (million)
1997	560.6		1.94
1998	589.5		1.98
1999	630.2		2.04
2000	640.3	100.0	2.11
2001		104.5	2.23
2002		120.3	2.31
2003		115.3	2.40
2004		132.1	2.45

i. Briefly discuss the likely difficulties that there will have been in determining the agricultural production indices?

ii. Combine the two production indices for agricultural production so that the resultant series has a common base.

iii. Produce a fixed base index series for agricultural production per head of population. Comment thoroughly upon the validity and interpretation of your results.

a.) **Laspeyre** **V** **Paasche**

	Laspeyre		Paasche
1.	Compares cost of buying base year quantities at current year prices with base prices.	1.	Compares cost of buying current year quantities at current year prices with year base year prices.
2.	Assumes that if prices had risen would still purchase same quantities as in base year. (Overestimates inflation)	2.	Assumes that if prices had risen would have bought same Quantities as in current year. (Underestimates inflation)
3.	Easy to calculate as weights fixed.	3.	Difficult, expensive and time consuming to keep re-calculating weights.
4.	Same basket of goods so different years are directly comparable.	4.	Difficult to make comparisons as changes in index reflect both changes in price and weights.

b.) i. - Collection of accurate data in reasonable time.
- Data subjectivity: includes what agricultural products to consider and what weights to use.
- Data updates : How often should the products and the weights be updated
- Index related problems:
  i. Methodology : Fixed , chained ,Paschee or Laspeyre.
  ii.Calculation : Is it possible to be calculated and how often.

ii. When you combine two indices with different base years you should use the most recent one as the base of the **combined** index.The bases appearing in the table are : Foodpro index 1970 and Farmfood index 2000.
Therefore the combined index base should be
Base 2000 = 100.
Now Index values of older bases are adjusted for the change in Base =
(index of older base ÷ index of common base)x100

Index value for 1997 = $\frac{560.6}{640.3}$x100 = 87.55

Index value for 1998 = $\frac{589.5}{640.3}$x100 = 92.066 i.e. 92.07

Index value for 1999 = $\frac{630.2}{640.3}$x100 = 92.42

For years 2000 to 2004 the values remain the same as 'Farmfood' index.

iii. To produce a fixed index per head of population:
1. Use the combined index from (ii.) and divide the values (year by year) by the corresponding values of the population.
   Example :
   In 1997 : 87.55/1.94 = 45.13
   In 2000 : 100/2.11 = 47.39
2. Take the resulting values and choose a suitable base to Rescale them: 2000 is the best base:
   Index value per head of population for 1997 :
   $=\frac{45.13}{47.39}\times 100$ =95.22
   Similarly : 98.11 for 1998;101.80 for 1999; 100 for 2000, 98.89 for 2001;109.89 for 2002;101.37 for 2003 and 113.77 for 2004.

5.(a) Define an Ideal Index, explain why it might be useful and state how you might calculate one.

(b) Workers in a company are from four different ethnic groups (labelled A,B,C and D). During a three year period the number of workers employed by the Emensy company in each group and their average weekly earnings (per person) are:

	Year 1		Year 2		Year 3	
Ethnic Group	Number	Earnings	Number	Earnings	Number	Earnings
A	182	505	225	531	232	584
B	103	245	66	268	71	293
C	7	908	9	873	18	821
D	45	125	55	133	60	143

Use suitable indices (Base year 1) for the three years to show the changes in each of the following:

i. Total earnings paid by Emensy,
ii. Average earnings for their workers as a whole, and
iii. The total number of workers employed.

(c) The following table shows the price of four daily newspapers sold in a particular city.

	Daily Planet	The Times	The News	The Globe
Year 1	90	65	80	80
Year 2	105	85	90	80
Year 3	120	90	105	90
Year 4	150	100	120	50

i. Find the simple aggregate price index and average price relative index for each year . (Use Year 1 as a base throughout).

ii. How could you improve the above analysis if your aim was to assess the general cost of newspapers in year 4 as opposed to year 1 ?

a) The over and the under estimation of price changes when using Laspeyre's and Paasche's indices led to the idea of ***Ideal*** index numbers. An ideal index should pass index tests that are used to determine how good an index is:

1. Time reversal test: Reversing the time subscripts produces the reciprocal of the original index:
Consider the index I_2 calculated for a period t_2 using a based period of t_1; the index I_1 calculated for the period t_1 using t_2 as base period is the reciprocal of I_1.
e.g. $I_2 = 2$ i.e. 200% then $I_1 = ½ = 0.5$ i.e. 50%

2. Factor reversal test:
The product of the price index and the quantity index should be equal to the index of total value:
Examples of the ideal indices are :

1. *Irving Fischer Index:*
The geometric mean of the original indices.

2.Marshall-Edgeworth Index :
Uses the arithmetic mean of the quantities purchased in the base and the current periods as weights.
In practice, the Marshall-Edgeworth and the Fischer indices give similar results.

PS : the only reason for providing a detailed answer is that this question was given in a previous LSE exam (2004) where 6 marks were allocated to it.
You should not answer in few lines a question that worth 6 marks and in 2 pages for a question that worth 2 marks.

b) i. Total earnings for Year 1 :
$\sum w_i p_1 =$ (182)(505)+(103)(245)+(7)(908)+(45)(125) = 129126
Total earnings for Year 2 :
$\sum w_i p_2 =$ (225)(531)+(66)(268)+(9)(873)+(55)(133) = 152335
Total earnings for Year 3 :
$\sum w_i p_3 =$ (232)(584)+(71)(293)+(18)(821)+(60)(143)= 179 649

Changes in total earnings: Choosing Year 1 as a base:
Index for Year 1 = 100

Index for Year 2 = $\frac{152335}{129126} \times 100$ **= 118**

Index for Year 3 = $\frac{179649}{129126} \times 100$ **= 139.1**

ii. Average earnings for Year 1 $= \frac{129126}{182+103+7+45} = \frac{129126}{337} = 383.16$

Average earnings for Year 2 $= \frac{152335}{225+66+9+55} = \frac{152335}{355} = 429.11$

Average earnings for Year 3 $= \frac{179\,649}{232+71+18+60} = \frac{179\,649}{381} = 471.52$

Changes in average earnings : choosing Year 1 as base :

Index for Year 1 = 100

Index for Year 2 = $\frac{429.11}{383.16}\times 100$ **= 111.99**

Index for Year 3 = $\frac{471.52}{383.16}\times 100$ **= 123.06**

iii. Total workers of Year 1 = 337

Total workers of Year 2 = 355

Total workers of Year 3 = 381

Changes in total workers: choosing Year 1 as base:

Index for Year 1 = 100

Index for Year 2 = $\frac{355}{337}\times 100$ **= 105.34**

Index for Year 3 = $\frac{381}{337}\times 100$ **= 113.06**

c) i. Simple aggregate index :

For Year 2 : $\frac{\sum_i^N p_{it}}{\sum_i^N p_{i0}}\times 100 = \frac{105+85+90+80}{90+65+80+80}\times 100 = \frac{360}{315}\times 100$ **=114.285**

Similarly, 128.57 for Year 3 and 133.33 for Year 4

The average price relative index :

For Year 2 : $\frac{1}{N}\sum_1^N \frac{p_{it}}{p_{i0}}\times 100 = \frac{1}{4}\left(\frac{105}{90}+\frac{85}{65}+\frac{90}{80}+\frac{80}{80}\right)\times 100$ **= 115**

Similarly, 128.89 for Year 3 and 133.26 for Year 4

ii. All you need here is to recognize that some of the weighting would be useful, for example, the quantity of each newspaper sold in each year.