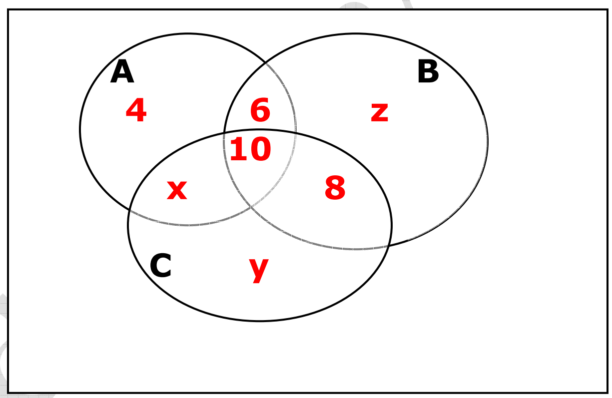
International Institute for Technology and Management

Tutoring Sheet #1

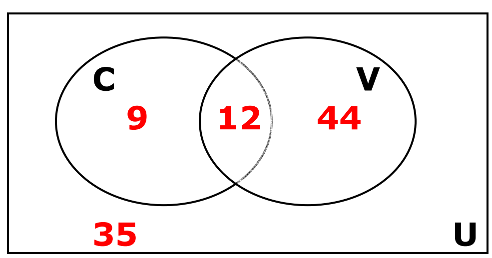
Unit 76 : Management Mathematics - Set Theory

1. The three sets A,B and C consist of the same number of elements ;Given that $(A\cap B\cap C) = 10$; $n(A\cup B\cup C) = 50$; $n(A\cap B) = 16$; $n(B\cap C) = 18$; $n(A\cap B^c\cap C^c) = 4$.
Calculate the order of each set.



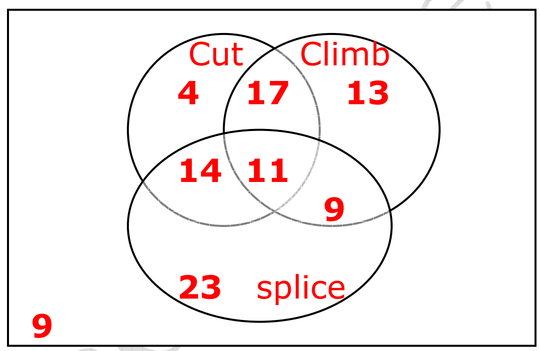
Let the missing regions be x,y and z :
$4+x+y+8+z+6+10=50$
$x+y+z=22$ -------(1)
Moreover,
$n(A)=n(B)=n(C)$
$4+x+10+6=z+6+10+8$
$=x+y+8+10$
$x+20 = z+24=x+y+18$
the last two:
$x+y = z+6$;substitute this in (1) :
$z+6+z=22$; $z = 8$;$n(B) = 6+10+8+z=24+8=32$
$n(A)=n(B)=n(C) = 32$.
There is no need for calculating x and y (x=12;y=2)

2. A researcher collecting data on 100 households found the following:21 have a computer; 56 have a VCR and 12 have both



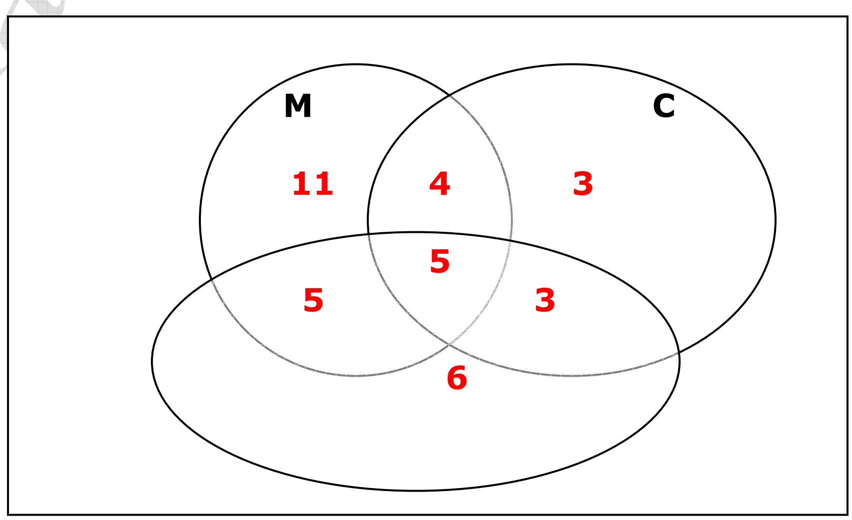
a. How many do not have a VCR = 9 + 35 = 44
b. How many have neither a computer nor a VCR
= 100 – (9+12+44) = 100 – 65 = 35
c. How many have a computer but not a VCR = 9

3. The employees in an electric utility cut down tall trees climb poles and splice wires. Out of 100 employees :
46 can cut tall trees; 50 can climb poles; 57 can splice wires.
28 can cut trees and climb poles;20 can climb poles and splice wires;25 can cut trees and splice wires;11 can do all three and 9 can not do any of the three (trainees)



a. How many can only cut tall trees = 4
b. How many can cut trees or climb poles
= 4+14+11+9+13+17= 68
or n(Cut U Climb) = n(Cut) + n(Climb) – n(Cut ∩ Climb)
n(Cut U Climb) = 46 + 50 – 28 = 68
c. How many can cut trees or climb poles or splice wires
4+14+23+9+13+17+11= 91

4. A computer technician repaired **30** computers of which:
25 required motherboards (MB);15 required CPU's;19 required Power Supply(PS);9 required both MB and CPU;10 required both MB and PS;8 required both CPU and PS;5 required all three.
a. Draw a Venn diagram for the above information.



b. Explain why his supervisor found the original information strange?
Since there are 37 computers while they should be 30 : 11+5+6+3+3+4+5 = 37 ??

c. Which piece of information had he given wrong if he did indeed repaired 30 computers. Explain why there is only one possible candidate for erroneous information and calculate the total number of parts that were used by him.

To determine the erroneous information, we look at the:
-Intersection of all 3 sets:which we usually decrease to decrease the total.
-Their intersection in pairs: which we increase to decrease the total.
- The order of the sets : n(M),n(P),n(C):which we decrease to decrease the total.

A possible erroneous information is $M \cap P^c \cap C^c$ (the part that lies only in M;its order is 11) since the other subsets are too small.

1. $M \cap P \cap C$ (of order 5) :if it is increased then the total number will increase which is not our goal.
   If it is decreased by x ,then the total decreases by x;
   However the maximum decrease is 5 which will not do
   Since 37 – 5 = 32 ;still exceeding 30 .
2. $M \cap P$ and $P \cap C$ we have to increase it to decrease the total ;no use since the decrease limit is ≤ 3 for C (the part that lies only in C) and ≤ 6 for S(the part that lies only in S).
3. $M \cap C$ (of order 9) then we have to increase it to decrease the total.However the size of decrease is limited to ≤3 from C ;
4. n(C) , n(P) if reduced ,still we have excess of 30.
   Limit for C is ≤ 3 ; limit for P ≤ 6.
5. The only option is to reduce n(M) by 7 to make the number of computers 30.

Therefore, 25 required motherboards (MB) should be
25 – 7 = 18 required motherboards (MB).
The part that lies only in M ($M \cap P^c \cap C^c$) becomes 4.
the total number of parts used :
4 + 3 + 6 +2(4+5+3) + 3(5) = 52

5 As a consequence of daily observations of a company, you discover the following facts about the sets O, S, W, I, P relating to the affairs of the company:

i) When the workforce are overworked(O) they always strike(S);
ii) The workforce are never overworked during the winter(W);
iii) The company's share price always increases(I) during the winter;
iv) The company's share price never increases when the workforce are on strike(S).
v) The workforce are never underpaid(P) when overworked.

(a) Convert each of the above facts into a mathematical statement about the appropriate sets.
(b) Draw a single Venn diagram to illustrate the above relationship between the sets.
(c) For each of the following statements indicate whether they are implied by the above facts:
A) No strikes occur in the winter
B) The share price never increases when the workforce are overpaid
C) The workforce never strike when they are underpaid in the winter.

(d) For each of the following subsets, describe its meaning in words and use a Venn diagram similar to (b) above to depict it:

$(S \cup P)^c$
$(S \cap P)^c$
$S \cap P^c \cap O^c$

a. **i.) $O \subseteq S$;**

ii.) $O \subseteq W^c$ or $W \subseteq O^c$ or $O \cap W = \phi$ or $n(O \cap W) = 0$

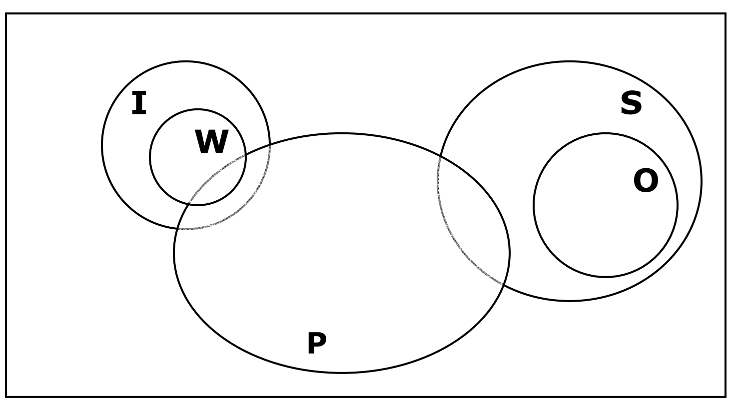
iii.) It may increase in other seasons : $W \subseteq I$

Note $I \subseteq W$ is completely wrong, it means the increase is *only* in winter.

iv.) $I \subseteq S^c$ or $S \subseteq I^c$ or $I \cap S = \phi$ or $n(I \cap S) = 0$

v.) $P \subseteq O^c$ or $O \subseteq P^c$ or $P \cap O = \phi$ or $n(P \cap O) = 0$

b. **O inside S**
O separate from W
W inside I
I separate to S
P separate to O



c. From diagram :
$W \cap S = \phi$

A is implied.
B not implied.
C is implied.

d. when answering such a question,never use union,complement, etc… will be marked wrong:

Remember Demorgan's:$(A\cap B)^c = A^c \cup B^c$;$(A\cup B)^c = A^c \cap B^c$

$(S \cup P)^c = S^c \cap P^c$: Workforce are not on Strike **and** workforce is not underpaid.

$(S\cap P)^c = S^c \cup P^c$: Workforce is not on strike **or** Workforce is not underpaid.

$S \cap P^c \cap O^c$ **:** Workforce are on Strike **and** work force are not Underpaid **and** Workforce is not overworked.

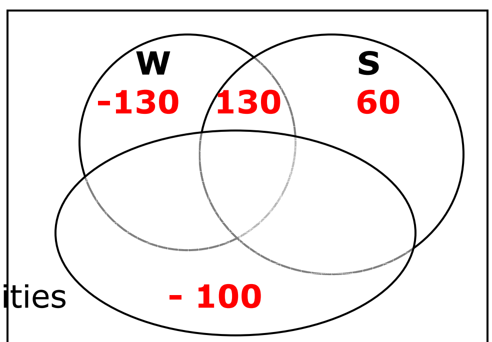
6 In order to study the computer awareness of its personnel a company assessed 500 of its workers. Each worker was tested to see if they were competent with Wordprocessing (W), Spreadsheets (S) or Database Systems (D). It was discovered that amongst the assessed workers there were 250 competent with W, 380 with S and 280 with D. 60 people were particularly capable as they were competent in all three computer skills. The survey further suggested that 190 workers were competent with S and D. A similar total of 190 workers were competent with S and W but as many as 250 were competent with both D and W.

i. Try to produce a Venn diagram for the above situation and hence show that there must be an error in the survey data.

ii. If one (and only one) of the above figures is incorrect determine which ones it could possibly be and, in each case, find a maximum and minimum value which the correct value might be.

iii. Show the following set on a Venn diagram, describe it in words and determine the maximum and minimum order of it:

$$W \cap (D^c \cup S)$$



i. It is obvious , in order to make **n(W)** = 250 ;we should have $n(W\cap S^c\cap D^c) = -130$ **??**
similarly $n(D\cap S^c\cap W^c) = -100$ **??**

ii. $n(W\cap S^c\cap D^c)$ **and** $n(D\cap S^c\cap W^c)$ need to be changed.The only possibilities are $n(W\cap S\cap D)$ **and** $n(W\cap D)$
The strategy is to let the order of one of these subsets to be x , Calculate the orders of all other affected subsets in terms of x and find the limits on the value of x (based on non-negativity of their order).If $n(W\cap S\cap D)$ were erroneous,you will find x = 190.
If $n(W\cap D)$ were erroneous, you will find $90 \le x \le 120$

iii. If $n(W\cap S\cap D)$ were erroneous,you will find its order = 190.
If $n(W\cap D)$ were erroneous, you will find its order $190 \le x \le 220$