

International Institute for Technology and Management



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Unit 76: Management Mathematics

Handout #5d

Difference Equations IV

Graphing Solutions

When graphing the solution of a difference equation Y_t is plotted against t , the result is **not** a smooth curve because t is allowed to take a **whole number values**. Consequently we join up the points in horizontal lines to create the staircase which more properly reflects the fact that t is discrete.

Example 1: the solution of the difference equation: $Y_t = 4Y_{t-1} + 21$ is given

by $Y_t = 8(4)^t - 7$

Suppose $t = 4$ then

$Y_t = 8(4)^4 - 7 = 2041$

You need to choose

The unit on the vertical

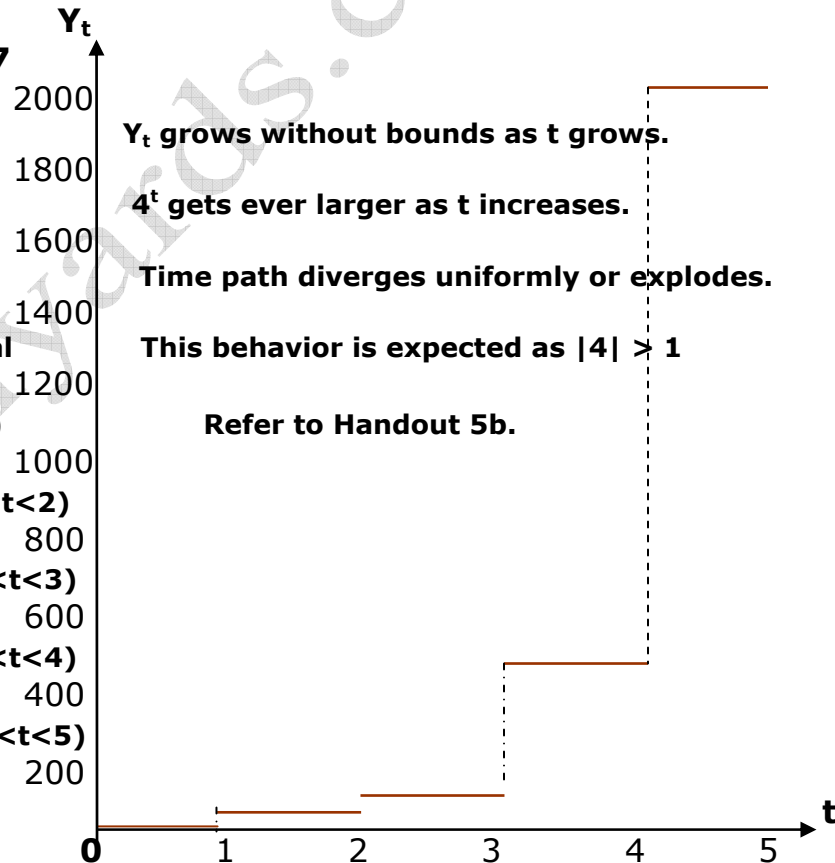
$t = 0; Y_t = 1$ ($0 < t < 1$)

$t = 1; Y_t = 25$ ($1 < t < 2$)

$t = 2; Y_t = 121$ ($2 < t < 3$)

$t = 3; Y_t = 505$ ($3 < t < 4$)

$t = 4; Y_t = 2041$ ($4 < t < 5$)



For comments, corrections, etc...Please contact Ahnaf Abbas: ahnaf@uaemath.com

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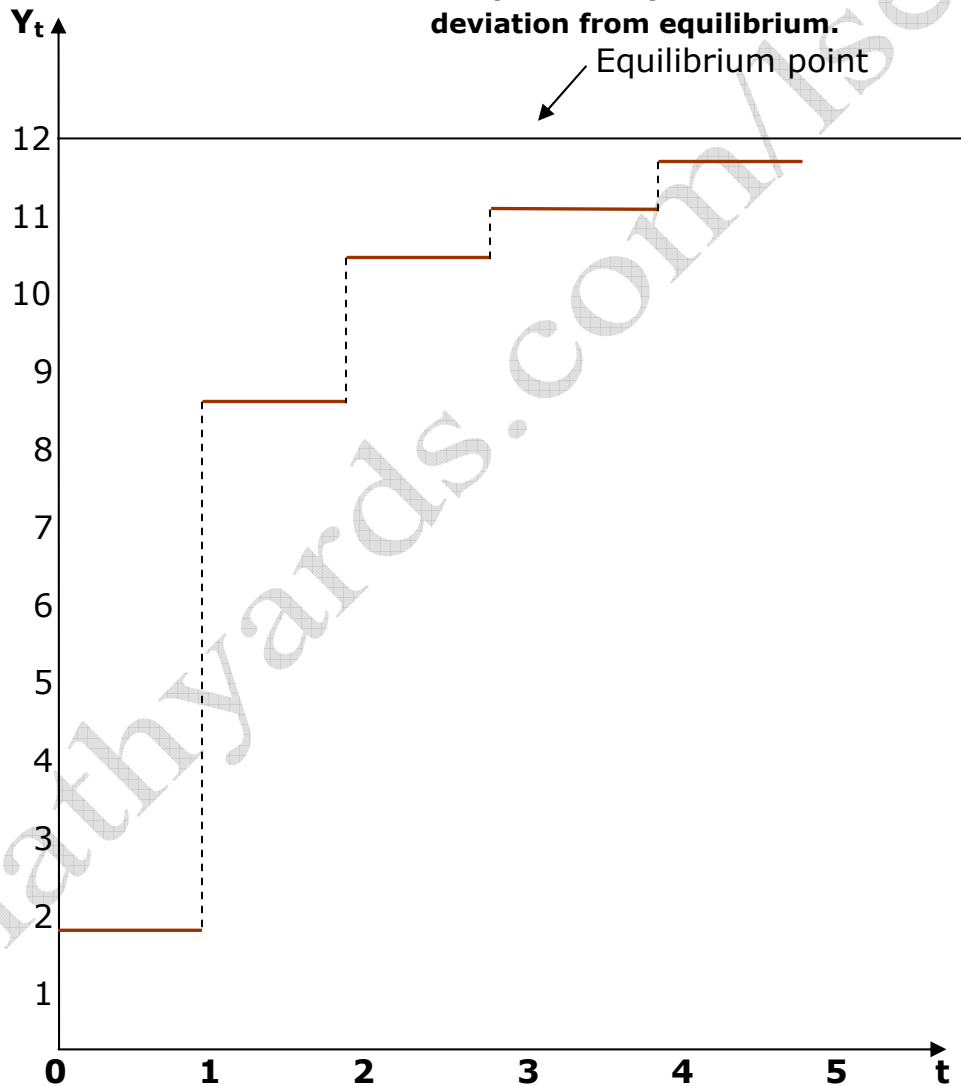
Example 2: the solution of the difference equation: $Y_t = \frac{1}{3}Y_{t-1} + 8$ is given

$$\text{by } Y_t = -10\left(\frac{1}{3}\right)^t + 12$$

t	0	1	2	3	4
Y_t	2	8.6	10.8	11.6	11.8

Recall that the particular solution is

the equilibrium value, where as the Complementary function measures the deviation from equilibrium.



Since $|1/3| < 1$, time path converges. (Refer to Handout 5b)

Y_t increases but eventually settle down at 12. We say that the time path converges uniformly to the value of 12 which is referred to as equilibrium point. This apparent since $(1/3)^t \rightarrow 0$ as $t \rightarrow +\infty$ so the complementary function disappears leaving just the particular value 12.