International Institute
for Technology and Management

Unit 76: Management Mathematics

Handout #2

Index Numbers

Study Guide pp 12 – 19

Topic	Interpretation
Introduction Index numbers are designed to measure the magnitude of economic changes over time. Briefly, this works in the following way. Suppose that an item's price is 75 in 1995. In 2002, an identical item cost 99. How has the price changed between 1995 and 2002?	Because they work in a similar way to percentages they make such changes easier to compare. The particular time period of 1995 which we've chosen to compare against, is called the ***base period***. The variable for that period, in this case the 75, is then given a value of 100, corresponding to 100%. The index can then be calculated for the later period of 2002 as a proportionate change as follows:
Simple price index (Single item) **Price relative =** $\frac{p_t}{p_0} \times 100$ **P_t : price in a later period** **P_0 : price in the base period**	Simple price index = $\frac{p_t}{p_0} \times 100 = \frac{99}{75} \times 100$ **= 132;** *The index number shows us that there has been a price increase of 32% since the base period.*

Simple Unweighted Aggregate Index

This is used for a fixed group of ***N*** items. Let p_{i0} be the price of the ith item in the base period. Let p_{it} be the price of this item in a second period : $\frac{\sum_i^N p_{it}}{\sum_i^N p_{i0}} \times 100$

Average Price relative index

It is obtained by calculating the average price of these items and calculating an index for these average prices.

$$\frac{1}{N} \sum_1^N \frac{p_{it}}{p_{i0}} \times 100$$

Disadvantage: *takes no account for quantities.*

Advantage: *index is independent of*

Example:

Item i	Qty	2000 Base Prices p_{i0}	2005 Prices p_{it}	p_t/p_0
A	1Kg	30	33	1.1
B	5Kg	15	24	1.6
Labour	6 hrs.	30	42	1.4
Total		75	99	4.1

Simple Aggregate price index: $\frac{\sum_i^N p_{it}}{\sum_i^N p_{i0}} \times 100$

$$= \frac{33+24+42}{30+15+30} \times 100 = \frac{99}{75} \times 100 = \mathbf{132}$$

Disadvantage: *quantities will remain the same throughout the analysis.*

Average Price relative index: $\frac{1}{N} \sum_1^N \frac{p_{it}}{p_{i0}} \times 100$

= (1/3)(1.1+1.6+1.4)x100 = 136.6

quantities.

Weighted Aggregate Price Index

The price is weighted by the quantity traded:
Called Index of total value

$$\frac{\sum_i^N p_{it} \times q_{it}}{\sum_i^N p_{i0} \times q_{i0}} \times 100$$

Take account of the different quantities for the items by multiplying the unit prices by the corresponding quantities.

The Expenditure Index :

$$\frac{\text{cost of party in 2005}}{\text{cost of party in 2003}} \times 100$$

Cost of 2003 party:
=(2.5)(25) + (4.5)(10)+(0.6)(10)
= 113.5

Cost of 2005 party:
=(3)(30) + (6)(8)+(0.84)(15)
= 150.6

Example: IITM Student's parties

Drink	Unit price (2003 party)	Qty (2003 party)	Unit price (2005 party)	Qty (2005 party)
	p_o	q_o	p_n	q_n
water	AED 2.50	25	AED 3	30
Juice	AED 4.50	10	AED 6.00	8
Soft	AED0.60	10	AED 0.84	15

The Expenditure Index : $\frac{\sum_i^N p_{it} \times q_{it}}{\sum_i^N p_{i0} \times q_{i0}} \times 100$

= (150.6/113.5) x 100 = 132.7

Laspeyre's Price Index
(Base period weighted)

It is a weighted aggregate price index where the ***quantities*** *are from the* ***base period****.*

$$\frac{\sum_i^N p_{it} \times q_{i0}}{\sum_i^N p_{i0} \times q_{i0}} \times 100$$

The Laspeyre's index, in effect, compares the total cost of purchasing the base-period quantities in the base period with what would have been the total cost of purchasing these same quantities in other periods.

Advantage: Laspeyre's price index uses quantity information from only the base period.This is valuable when it is difficult to obtain that information for every year.

The Laspeyre's index for the IITM e.g.

$\sum_i^N p_{it} \times q_{i0}$ =(3x25)+(6x10)+(0.84x10) =143.4

$\sum_i^N p_{i0} \times q_{i0}$ =(2.5x25)+(4.5x10)+(0.6x10)=113.5

$$\frac{\sum_i^N p_{it} \times q_{i0}}{\sum_i^N p_{i0} \times q_{i0}} \times 100 = \frac{143.4}{113.5} \times 100 = 126.3$$

Disadvantage: This could be a disadvantage if the base period quantities were not representative of the time series being considered.Thus becoming outdated.

One way around this problem is to construct moving Laspeyre's price index in which the base period is changed from time to time through the acquisition of quantity information for new base periods. The Consumer Price Index(CPI) is constructed in essentially this way.

Paasche's Price index
(end year or Current period weighted)

Calculated based on the amount spent on each item in the current year at the ***base period prices***.

$$\frac{\sum_{i}^{N} p_{it} \times q_{it}}{\sum_{i}^{N} p_{i0} \times q_{it}} \times 100$$

The Paasche's index for the IITM e.g.

$\sum_{i}^{N} p_{it} \times q_{it}$ = (3 x 30) + (6 x 8) + (0.84 x 15)

=150.6

$\sum_{i}^{N} p_{i0} \times q_{it}$ =(2.5x30)+ (4.5 x 8) + (0.6 x 15)

=120

$$\frac{\sum_{i}^{N} p_{it} \times q_{it}}{\sum_{i}^{N} p_{i0} \times q_{it}} \times 100 = \frac{150.6}{120} \times 100 = 125.5$$

Laspeyre's (Aggregate) volume Index

The prices remain relatively stable and it is the quantities of items which are changing.

$$\frac{\sum_{i}^{N} p_{i0} \times q_{it}}{\sum_{i}^{N} p_{i0} \times q_{i0}} \times 100$$

Paasche's (Aggregate) volume Index

$$\frac{\sum_{i}^{N} p_{it} \times q_{it}}{\sum_{i}^{N} p_{it} \times q_{i0}} \times 100$$

Work the IITM example ; you should get:

Laspeyre's volume index is 105.7
Paasche's volume index is 105.0

Laspeyre's Index	**Paasche's Index**
Advantages	**Advantages**
1. Weights need calculating only once. 2. The calculation of the index is faster.	Weights are up-to-date and more relevant.
Disadvantages	**Disadvantages**
1. Base weights quickly become irrelevant. 2. Index tends to overstate the price increase since the weights are not altered to allow movement from expensive items to cheaper ones.	1. Collection of data to calculate latest weights may prove difficult. 2. Prices changes are under estimated at the time of rising prices.

Laspeyre's vs. Paasche's

Ideal Indices

The over and the under estimation of price changes when using Laspeyre's and Paasche's indices led to the idea of ***Ideal*** index numbers. Two of these are :

1. ***Irving Fischer Index:***
The geometric mean of the original indices.

$$\sqrt{\frac{\sum_i^N p_{it} \times q_{i0}}{\sum_i^N p_{i0} \times q_{i0}} \times \frac{\sum_i^N p_{it} \times q_{it}}{\sum_i^N p_{i0} \times q_{it}}} \times 100$$

2.Marshall-Edgeworth Index :

Uses the arithmetic mean of the quantities purchased in the base and the current periods as weights.

$$\frac{\sum_i^N p_{it} \times \left(\frac{q_{it} + q_{i0}}{2}\right)}{\sum_i^N p_{i0} \times \left(\frac{q_{it} + q_{i0}}{2}\right)} \times 100$$

In practice, the Marshall-Edgeworth and the Fischer indices give similar results.

Index Tests

Are used to determine how good an index is:

1. Time reversal test:
Reversing the time subscripts produces the reciprocal of the original index.

2. Factor reversal test:
The product of the price index and the quantity index should be equal to the index of total value.

1. Consider the index I_2 calculated for a period t_2 using a based period of t_1; the index I_1 calculated for the period t_1 using t_2 as base period is the reciprocal of I_1. e.g. $I_2 = 2$ i.e. 200% then $I_1 = ½ = 0.5$ i.e. 50%

2. $$\left(\frac{\sum_i^N p_{it} \times q_{it}}{\sum_i^N p_{i0} \times q_{it}} \times 100\right)\left(\frac{\sum_i^N p_{i0} \times q_{it}}{\sum_i^N p_{i0} \times q_{i0}} \times 100\right)$$

$$= \frac{\sum_i^N p_{it} \times q_{it}}{\sum_i^N p_{i0} \times q_{i0}} \times 100$$

Chain-Linked Index Numbers

When base period is updated regularly.
Calculated by using a previous base period as a base.

Laspeyre's and Paasche's Chain Price indices:

$$\frac{\sum_i^N p_{it} \times q_{i0}}{\sum_i^N p_{i\,t-1} \times q_{i0}} \times 100 \; ; \; \frac{\sum_i^N p_{it} \times q_{it}}{\sum_i^N p_{i\,t-1} \times q_{it}} \times 100$$

For chain un-Linked index :Index in year t = (Price in year t)/(price in year t-1)x100