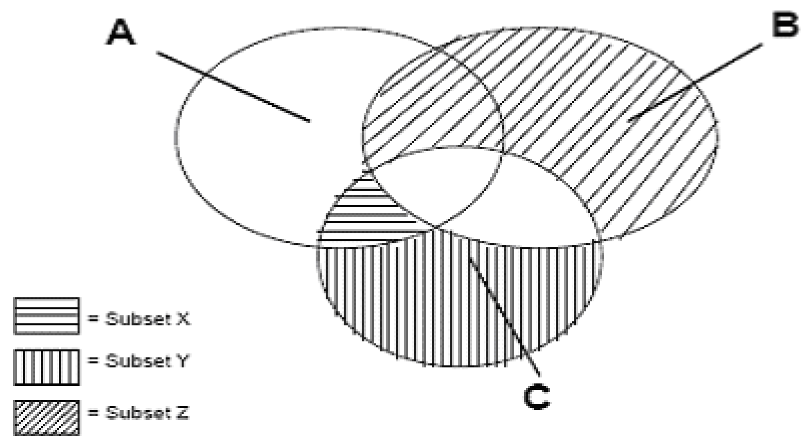
International Institute for
Technology and Management

October 28, 2007 **Duration: 90 minutes**

Unit 76: Management Mathematics Assignment – 1

Answer all the following questions:

1. Look at the following normal type of Venn diagram of sets A, B and C.



i) Write the set notation definitions of each of the 3 different shaded subsets (X, Y and Z) in terms of A, B and C (as simply as possible). Also produce a set definition for the combined unshaded region. (5 marks)

$X = A \cap C \cap B^c$
$Y = C \cap A^c \cap B^c$
$Z = B \cap C^C$
Unshaded region : $(B \cap C) \cup (A \cap B^c \cap C^c)$

ii) Draw another Venn diagram with three overlapping sets A, B and C and indicate the following subsets upon it:

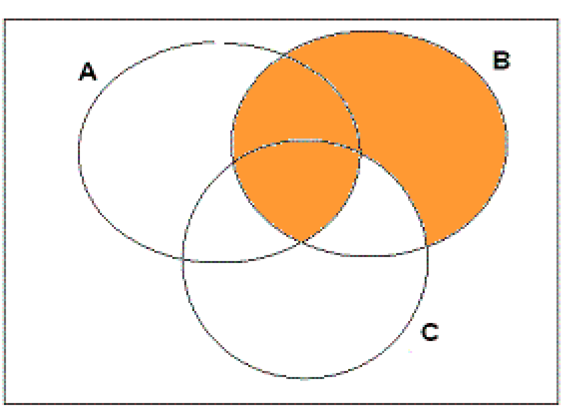
I) $(A \cap B) \cup (B \cap \overline{C})$

II) $\overline{A} \cup \overline{B} \cup C$

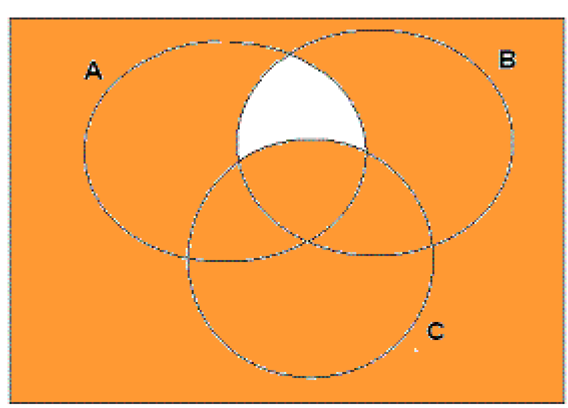
III) $(\overline{A \cup B}) \cap \overline{C}$

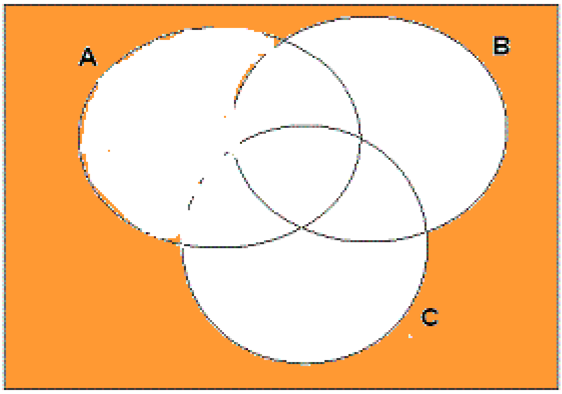
IV) $(A \cup B) \cap (B \cup C)$ (5 marks)

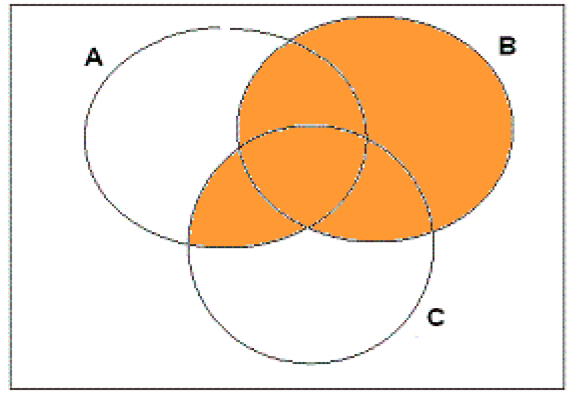
i)



ii)



iii) 

iv) 

iii) If the given Venn diagram denotes the percentages of a population that may or may not have the attributes A, B or C and $n(A) = 50\%$, $n(B) = 30\%$, $n(C) = 40\%$, $n(A \cap B) = 15\%$, $n(B \cup C) = 62\%$, $n(A \cup C) = 70\%$, determine the maximum and minimum orders (%) of the following subsets:

I) $n(A \cap B \cap C)$

II) $n(\overline{A} \cap \overline{B} \cap \overline{C})$ (10 marks)

$n(A) = 50$, $n(B) = 30$, $n(C) = 40$, $n(A \cap B) = 15$

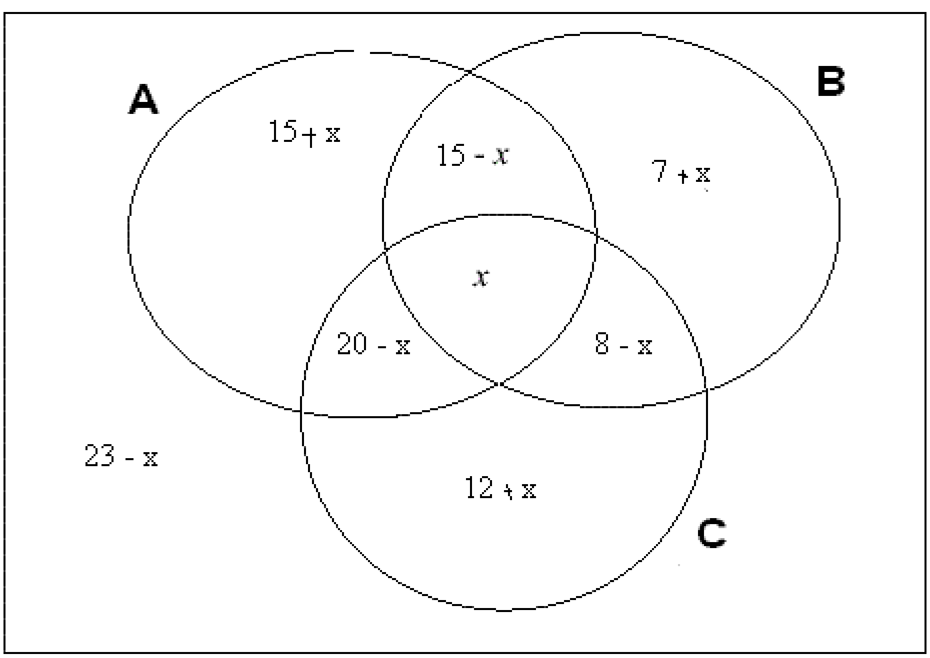
$n(B \cup C) = 62$, $n(A \cup C) = 70$

$n(B \cup C) = n(B) + n(C) - n(B \cap C)$

$\Rightarrow$ $n(B \cap C) = 30+40 - 62 = 8$

$n(A \cup C) = n(A) + n(C) - n(A \cap C)$

$\Rightarrow$ $n(A \cap C) = 50+40 - 70 = 20$



i) $15 + x > 0 \Rightarrow x > -15$

$15 - x > 0 \Rightarrow x < 15$

$7 + x > 0 \Rightarrow x > -7$

$x > 0$

$20 - x > 0 \Rightarrow x < 20$

$8 - x > 0 \Rightarrow x < 8$

$12 + x > 0 \Rightarrow x > -12$

$23 - x > 0 \Rightarrow x < 23$

Ignore all negative values : $0 < x < 8$

ii) $n(A^c \cap B^c \cap C^c) = 23 - x$ and $0 < x < 8$

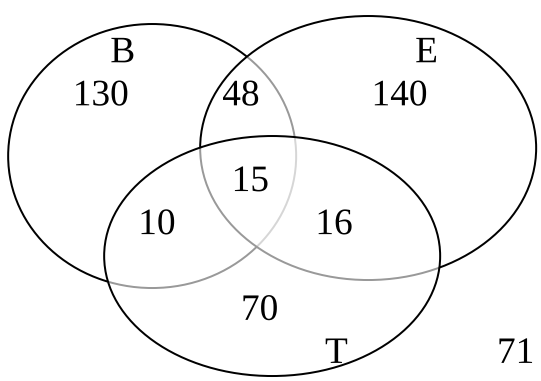
$-8 < x < 0 \Rightarrow 23 - 8 < 23 - x < 23 - 0 \Rightarrow 15 < 23 - x < 23$

Minimum order = 23 – 8 =15 , Maximum order = 23 - 0 = 23

2. In 2007, a company's total fleet of cars (500 in total) undergoes rigorous testing and each might fail because of faults in one or more of the following categories: Brakes (B), Exhaust (E) or Tyres (T). The following data shows the number of cars with certain types of faults when tested:

Fault(s)	Number of Cars Failed in 2007
B	203
E	219
T	111
B and T	25
B and E	63
T and E	31
B, T and E	15

i. Draw a suitable fully annotated Venn diagram to illustrate the above information and determine the number of fleet cars that showed no faults. **(4 marks)**



From the diagram : 71 fleet cars showed no faults.

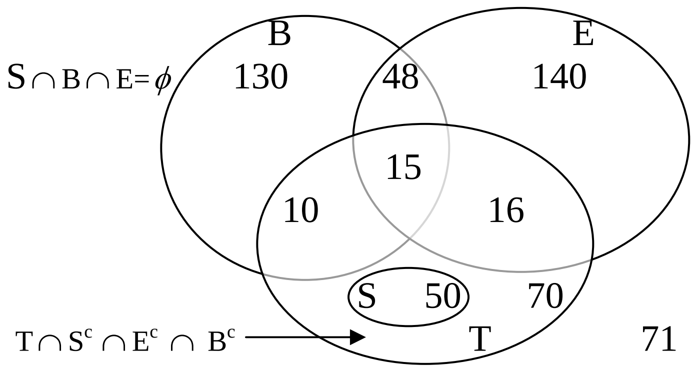
ii. It is later discovered that faulty steering (S), which always results in the car having faulty tyres, affected 10% of the company's car fleet in 2007. Redraw your Venn diagram to include information about (S) and determine the minimum and maximum number of cars that have no faults other than faulty tyres not due to faulty steering. **(5 marks)**

Faulty steering (S) , which always results in the car having faulty tyres $\Rightarrow$ $S \subset T$

n(S) = 10% of 500 = 50

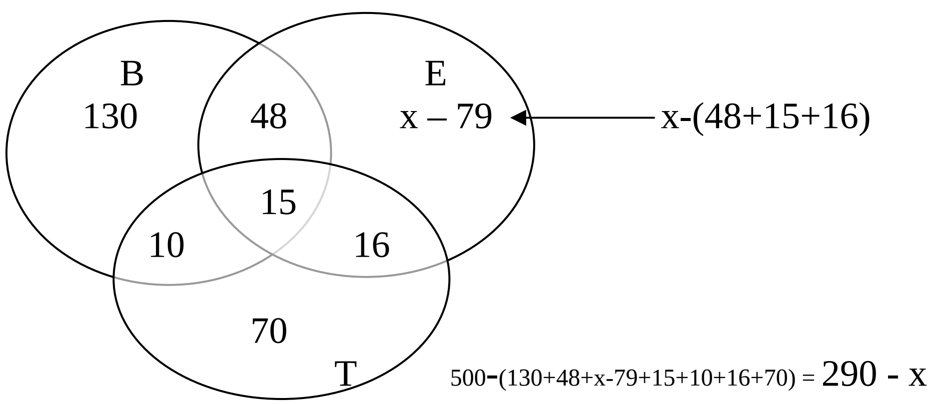
Faulty tyres not due to faulty steering : $T \cap S^c$

No other faults : $T \cap S^c \cap E^c \cap B^c$



$n(T \cap (S \cup E \cup B)^c) = 111 - (10 + 15 + 16 + 50) = 20$ Minimum <u>or</u> $70 - 50 = 20$

iii. The company's car maintenance manager has subsequently asserted that the stated number of Exhaust failures (currently thought to be 219) is wrong. Assuming this is the only error in the tabulated data above, what are the minimum and maximum possible number of Exhaust failures n(E)? **(5 marks)**



Since n(E)= 219 is the only error , the only affected areas are : $E \cap B^c \cap T^c$ (E only)

and the area outside the three sets : $E^c \cap B^c \cap T^c$

let n(E) = x $\Rightarrow$ $n(E \cap B^c \cap T^c) = x-(48+15+16) = x - 79$

$x - 79 \geq 0 \Rightarrow x \geq 79$, minimum n(E) = 79

$290 - x \geq 0 \Rightarrow x \leq 290$, maximum n(E) = 290

iv. Assuming n(E) is indeed 219, state the meaning of (and determine the number of cars in) each of the following subsets:

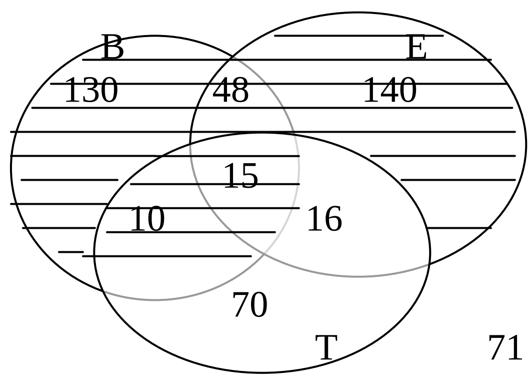
I) $(E \cap T^c) \cup B$

II) $(B \cup T) \cap S^c$

Indicate each of these subsets on an appropriate Venn diagram.

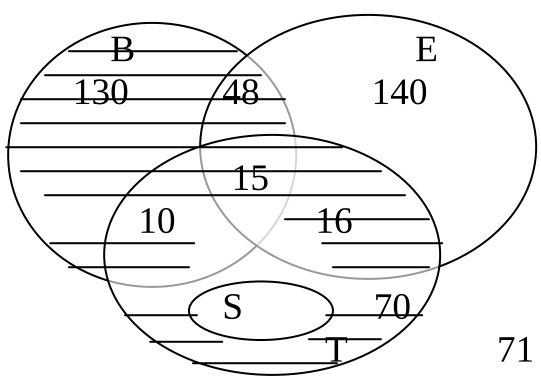
(6 marks)

I) $(E \cap T^c) \cup B$: Cars having exhaust faulty but **not** tyre faulty **or** cars with brake faulty.



$n[(E \cap T^c) \cup B] = 130 + 48 + 140 + 25 = 343$

II) $(B \cup T) \cap S^c$: cars that are either with brake faulty or tyre faulty and not with steering faulty.



$n(B \cup T) = n(B) + n(T) - n(B \cap T)$

$n[(B \cup T) \cap S^c] = n(B) + n(T) - n(B \cap T) - n(S)$

$= 203 + 211 - 25 - 50 = 339$

END of QUESTIONS