International Institute for
Technology and Management

Unit 05a:Mathematics 1 **Tutoring Sheet #13**

ANSWERS

1. Find the total cost function if the marginal cost is

$q + 5q^2 + e^q$ and the fixed cost is 10 .

MC = $q + 5q^2 + e^q$; FC = 10 ; TC = ?

$$TC = \int MC\, dq = \int \left(q + 5q^2 + e^q\right) dq = \frac{q^2}{2} + \frac{5q^3}{3} + e^q + C$$

C is not 10, to find C:

FC = 10 = TC(0)= 0 + 0 + e^0 + C

$10 = C + 1 \Rightarrow C = 9 \quad \therefore TC = \frac{q^2}{2} + \frac{5q^3}{3} + e^q + 9$

2. A company's marginal cost function is :

$MC = 32 + 18q - 12q^2$ and fixed costs of 43. Find the firm's total cost function, average cost function and variable cost.

$MC = 32 + 18q - 12q^2$; FC = 43;

TC =? ; AC =? ; VC =?

$$TC = \int MC\, dq = \int \left(32 + 18q - 12q^2\right) dq = 32q + \frac{18q^2}{2} - \frac{12q^3}{3} + C$$

$TC = 32q + 9q^2 - 4q^3 + C$; to find C :

$FC = 43 = TC(0) = 0 + 0 + 0 + C \Rightarrow C = 43$

$TC = 32q + 9q^2 - 4q^3 + 43$

$$AC = \frac{TC}{q} = \frac{32q + 9q^2 - 4q^3 + 43}{q} = 32 + 9q - 4q^2 + \frac{43}{q}$$

$TC = VC + FC \Rightarrow$

$VC = TC - FC = 32q + 9q^2 - 4q^3 + 43 - 43$

$VC = 32q + 9q^2 - 4q^3$

3. A firm's marginal cost function is :

$\frac{20}{\sqrt{q}}e^{\sqrt{q}} + q^3 + \frac{1}{q+1}$ and fixed costs of 20.

Determine the total cost function.

TC = $\int MC\,dq = \int\left(\frac{20}{\sqrt{q}}e^{\sqrt{q}} + q^3 + \frac{1}{q+1}\right)dq$

To determine the integral of $\frac{e^{\sqrt{q}}}{q}$,use the substitution u = $\sqrt{q}$

$\Rightarrow$ du = $\frac{1}{2\sqrt{q}}\mathrm{dq}$ $\Rightarrow$ $\int \frac{20e^{\sqrt{q}}}{\sqrt{q}}dq$ =20$\int 2e^u du$ =40e^u + C

TC = 40$e^{\sqrt{q}}$ + $\frac{q^4}{4} + \ln|1+q|$ + C

FC = 20 = TC(0) =40e^0 + 0 + ln1 + C (Note:ln1 = 0)

$\Rightarrow$ 20 = 40 + C $\Rightarrow$ C = - 20

TC = 40$e^{\sqrt{q}}$ + $\frac{q^4}{4} + \ln(1+q)$ - 20

4. The marginal cost is a function of output as follows :

MC = 10 – q + q^2

Determine the extra cost which is incurred when production is increased from 2 to 4.

Here we need to find TC(4) – TC(2)

Since TC = $\int MC\,dq$ $\Rightarrow$ TC(4) – TC(2) = $\int_2^4 MC\,dq$

= $\int_2^4 (10 - q + q^2)dq$ = **10q -** $\frac{q^2}{2} + \frac{q^3}{3}\Big|_2^4$

= $\left(10(4) - \frac{4^2}{2} + \frac{4^3}{3}\right) - \left(10(2) - \frac{2^2}{2} + \frac{2^3}{3}\right) = \frac{98}{3}$

5. A company produces only product X.When producing q units the Marginal cost is given by:

MC =1 - $\frac{1}{(q+1)^2}$ if the ***average cost*** per unit when producing **4 units is 3.05**.what is the total cost of producing 5 units of X?

Total cost of producing 5 units = TC(5)

We know TC = $\int MC\,dq$ = $\int\left(1-\frac{1}{(q+1)^2}\right)dq$

TC = $\int\left(1-(q+1)^{-2}\right)dq = q+(q+1)^{-1}+C = q+\frac{1}{q+1}+C$

We need to find C ? The fixed cost is not given!

We are given that the average cost of producing 4 units is 3.05

AC(4) = 3.05 $\Rightarrow$ TC = q AC $\Rightarrow$ **TC(4)** = 4 AC(4)=4(3.05)=**12.2**

But TC = $q+\frac{1}{q+1}+C$ $\Rightarrow$ TC(4) = 4 + $\frac{1}{5}$ +C =12.2

$\Rightarrow$ C = 8 $\Rightarrow$ TC = $q+\frac{1}{q+1}+8$

Now TC(5) = 5 + $\frac{1}{6}+8$ = $\frac{79}{6}$

6. The marginal revenue for a commodity is given by :

MR = 10 – 2q^2 and the total cost is TC = q^2 + 4q + 2

where q is the number of units produced.Find the revenue function and determine the profit function.

TR = $\int MR\,dq$ = $\int\left(10-2q^2\right)dq = 10q-\frac{2q^3}{3}+C$

To find C ? what is the revenue from selling 0 items? Of course it is 0 ie TR(0) = 0 hence TR = $10q-\frac{2q^3}{3}$

The profit function: $\prod = TR-TC = 10q-\frac{2q^3}{3}-q^2-4q-2$

$\prod = -\frac{2q^3}{3}-q^2+6q-2$