



November 29th, 2007

Unit: 05a – Mathematics 1

GROUP(B)-VERSION B

This paper is not to be removed from the Examination Halls

Student Name :

Student Number :

Tuesday 27th November 13 : 30 pm – 15 : 30 pm

Candidates should answer **NINE** of the following **ELEVEN** questions: **SEVEN** from section A (60 marks in total) and **TWO** from section B (20 marks each).

Candidates are strongly advised to divide their time accordingly.

Graph paper is provided at the student request.

Calculators **May NOT** be used for this paper.

PLEASE TURN OVER

SECTION A

Answer all **SEVEN** questions from this section (60 marks in total)

1. The supply equation for a good is

$$q = p^2 - 2p + 3$$

and the demand equation is

$$q = -2p^2 + p + 9$$

where p is the price.

Sketch the supply and the demand functions for $p \geq 0$

Determine the equilibrium price and quantity.

2. Find the maximum value of the function:

$$f(x) = (x^2 - x + 1)e^{-x}$$

Verify that it is a maximum.

3. Determine the following integrals

$$\int \frac{(1 - \ln x)^2}{x} dx, \quad \int \sin^3 x \cos^5 x dx$$

4. The marginal cost for a company is

$$4q^3 + 6q + e^q - 1$$

and fixed costs of 60 .

Find the total cost, the variable cost and the average cost functions.

5. (a) A firm's demand function is

$$p = aq + b \quad (a < 0 ; b > 0)$$

Fixed costs are c and variable costs are d per unit.

Show that the profit is maximized when $q = \frac{d-b}{2a}$

6. A firm has average variable cost

$$q^2 + 7q + \frac{\ln(q^3 + 7)}{q}$$

and fixed costs of 7 . Find the total cost function and the marginal cost function.

7. Determine the following integrals

$$\int \frac{1}{x\sqrt{\ln x}(\ln x + 4\sqrt{\ln x} + 4)} dx, \quad \int \frac{\sqrt{1 + \tan x}}{\cos^2 x} dx$$

SECTION B

Answer **TWO** questions from this section (20 marks each)

8. (a) A firm is a monopoly for the good it produces, It has a marginal cost function $MC = 6q^2 + 8$ and fixed costs of **20**. The demand equation for its good is given by $p + 2q = 40$ where p is the price. Find expressions in terms of q , for the total revenue and profit. Determine the value of q that maximises the profit. Calculate this maximum profit.

(b) Determine the following integrals

$$\int x^2 \sqrt{x^3 + 1} dx \quad , \quad \int \frac{x}{\sqrt{x+1}} dx$$

9. (a) A firm is a monopoly its fixed costs are 20 it has average variable cost function $AVC = 10 + q$ where q denotes its production level , the demand function of the good

$$\text{produced by firm is } q = 10 - \frac{p}{2}$$

where p is the price. Find expressions, in terms of q , for the revenue and the profit and determine the value of q that maximizes the profit.

- (b) The function $f(x) = x^2 - \ln(\sqrt{2}x)$ is defined for $x > 0$
Determine the critical points of f and specify their nature.

- 10.(a) A firm's marginal revenue function is $MR = 11 - q$
The firm's marginal cost function is
 $MC = q^2 - 3q + 3$
where q is either the quantity sold or produced.
Find the profit-maximizing level of output and verify that it is a maximum.

(b) Determine the following integrals

$$\int \frac{x+1}{(x^2 + 2x + 5)^2} dx \quad , \quad \int \frac{e^{-x} - e^x}{e^x + e^{-x}} dx$$

11. A firm faces a total cost function $TC = q^3 - 4q^2 + 12q$
- Determine the firm's average cost (AC) and marginal cost (MC) functions.
 - Sketch the average cost (AC) and the marginal cost (MC) on the same graph.
 - If price is \$ 15 , which level of output will a profit maximising firm choose?

END OF PAPER