International Institute for
Technology and Management

Unit 05a: Mathematics 1 **Tutoring Sheet #14**

REVISION I

1. The functions f(x) and g(x) are given by :

$f(x) = 4x^2 - 8x - 1$, $g(x) = -4x^2 - 2x - 1$

Sketch the graphs of y = f(x) and y = g(x) for x > 0 on the same diagram, and determine the positive value of x at which these two graphs intersect.

2. The supply equation for a good is $q = p^2 + 7p - 2$ and the demand equation is $q = -p^2 - p + 40$ where p is the price. Sketch the supply and the demand functions for $p \geq 0$ Determine the equilibrium price and quantity.

3. A firm's cost function is C = 20q + 60 and the revenue is $R = q^2 - 8q$. Sketch the graphs of C and R on the same diagram .Find the break even value of q.

4. A monopolist's average cost function is given by :

$$9 + \frac{3}{10}q + \frac{30}{q}$$

Where q is the quantity produced, the demand function for the good is $q = 40 - \frac{4}{3}p$

Determine expressions, in terms of q , for the revenue and the profit and determine the value of q that maximizes the profit. Find the maximum profit.

5. Find the maximum value of the following functions(show it's maximum):

a. $f(x) = (1+x)e^{\frac{-x}{2}}$ b. $f(x) = x - x \ln x$

c. $f(x) = xe^{-3x} - 2$ d. $f(x) = -\sqrt{x^2+1}$

6. Find the minimum value of the following functions(show it's minimum) :

a. f(x) = 2x – ln x b. f(x) = $x^2 - \ln(\sqrt{2}\,x)$

c. f(x) = $e^{\mathbf{x}} + e^{\mathbf{-x}}$ d. f(x) = $x^{\mathbf{2}}$ -2x + 5

7. A firm has average variable cost :

$$q^2 + q + \frac{e^q}{q} - \frac{1}{q}$$

and fixed costs of 11.Find the total cost function and the marginal cost function.

8. The marginal cost for a company is $2q^3 + 6q + e^{0.5q} - 5$ and fixed costs of 65 .Find the total cost ,the variable cost and the average cost functions.

9. A firm has marginal cost 1 + q Determine by how much its total cost function is increased if its production is raised from From 2 to 4 units.

10. The marginal revenue from a product is given by $\dfrac{40}{e^{0.5q}} + 10$

Find the demand function for the product.

11. Determine the integrals :

a. $\int (6x^{-3} + 4x^{-1})\,dx$ b. $\int \left(\frac{3}{x} + e^{-4x}\right) dx$ c. $\int (4x+5)^7 dx$

d. $\int (2q^3 - 6q - e^{3q} - 5)dq$ e. $\int \frac{\sqrt{\ln x}}{x}\,dx$

f. $\int (x+1)e^{x^2+2x}dx$ g. $\int \frac{x+3}{(x^2+6x-7)^2}\,dx$

h. $\int x^3\sqrt{x^2+2}\,dx$ i. $\int x^2 e^x\,dx$ j. $\int x^2\sqrt{x+3}\,dx$

k. $\int \frac{\ln x}{x^2}\,dx$ l. $\int \frac{x+1}{x^2+2x+8}\,dx$ m. $\int \frac{x+3}{x^2+4x+5}\,dx$

12. Calculate the derivatives of the following functions:

a. $\left(x^2+3\right)^{\frac{1}{3}}$

b. $\sqrt{x+6}$

c. $\dfrac{x^2+2}{x^3+x}$

d. $\ln(x^2 + 3)$

e. $-e^{-2x}$

f. $(1-2x)e^{x^2+3}$

g. $x^2 \ln(x^3+1)$

h. $\dfrac{e^x+e^{-x}}{x}$

i. $x^2 - e^{\sqrt{2}x}$

j. $\dfrac{1-\ln x}{1+\ln x}$

13. Assume the profit (π) of an electricity generation company can be expressed as a function of output as follows:

$\pi = -40 + 140q - 10q^2$

a. Compute the profit-maximizing level of output and verify it is a maximum.

b. Compute the level of profits for this level of output.

14. A firm's total costs are given by the following expression:

$TC = \dfrac{1}{3}q^3 - 5q^2 + 30q$

a. Derive an expression for the firm's average cost (AC) function.
b. Find the output level at which the firm's average cost is at a minimum. Verify that it is a minimum.
c. What is the value of average costs at this level of output?
d. Derive an expression for the firm's marginal cost (MC)
e. Assume this firm operates in a perfectly competitive market and is able to sell its output at a price of £14 per unit. Determine its profit-maximising level of output.