International Institute for
Technology and Management

Unit : 05a - Mathematics 1

Handout #9

Integration I: Anthony & Biggs pp: 317 - 329 ,330-332

Topic	Interpretation
Anti-Derivative F′(x) = f(x) Example: f(x)=2x is the derivative of F(x)= x^2 here,F′(x) = f(x) F(x) = x^2+1 ;F′(x) = 2x	F(x) is the antiderivative of f or the primitive of f . x^2 is the antiderivative of 2x x^2+1 is the antiderivative of 2x All antiderivatives of a certain function differ only by a constant.
Indefinite Integral $\int f(x)dx$ Example: $\int 2xdx = x^2 + C$	f(x) is the derivative of what function? Derivative of $x^2 + C$ is 2x
Basic rules: $\int x^k dx = \dfrac{x^{k+1}}{k+1} + C \quad ; \; k \neq 1$ Example: $\int x^5 dx$ $\int \sqrt{x}dx = \int x^{\frac{1}{2}}dx$ *Things to Remember:* $\int kdx = kx + C$ $\int kf(x)dx = k\int f(x)dx$	$= \dfrac{x^6}{6} + C$ $= \dfrac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \dfrac{x^{\frac{3}{2}}}{\frac{3}{2}} + C = \dfrac{2x^{\frac{3}{2}}}{3} + C$ Examples: $\int 5dx = 5x + C \quad ; \quad \int dx = x + C$ $\int 4x^2 dx = 4\int x^2 dx = 4\dfrac{x^3}{3} + C$

<table>
<tr>
<td>Definite Integral
$\int_a^b f(x)dx = F(b) - F(a)$
Example: $\int_0^1 x^4 dx$</td>
<td>$\int_0^1 x^4 dx = \frac{x^5}{5}\Big|_0^1$ = F(1) – F(0)
$= \frac{1^5}{5} - \frac{0^5}{5} = \frac{1}{5}$</td>
</tr>
<tr>
<td>Integration by Substitution

Suppose you need to find :

$\int (2x+1)^2 dx$</td>
<td>One way to do it is to expand $(2x+1)^2$
So it will turn into something we know based on the above rule:
$(2x+1)^2 = 4x^2 + 4x + 1$
$\int (2x+1)^2 dx = \int (4x^2 + 4x + 1)dx$
$= 4\frac{x^3}{3} + 4\frac{x^2}{2} + x + C = \frac{4x^3}{3} + 2x^2 + x + C$</td>
</tr>
<tr>
<td rowspan="2">What about : $\int (2x+1)^{12} dx$
It is not practical to expand $(2x+1)^{12}$;
So we use Substitution.
<u>Example:</u> $\int x^3(\sqrt{x^2+1})dx$
Let $u = x^2 + 1$,then the integral becomes : $\int x^3\sqrt{u}dx$
We need to change *dx* into *du*
$u = x^2 + 1 \Rightarrow du = 2x\, dx$
$\Rightarrow dx = \frac{du}{2x}$ substituting this in the above integral:
$\int x^3\sqrt{u}\frac{du}{2x} = \frac{1}{2}\int x^2\sqrt{u}du$
Now, we need to get rid of x^2 :
$u = x^2 + 1 \Rightarrow x^2 = u - 1$
$\frac{1}{2}\int x^2\sqrt{u}du = \frac{1}{2}\int (u-1)\sqrt{u}du$
$= \frac{1}{2}\int u\sqrt{u}du - \frac{1}{2}\int \sqrt{u}du$
$= \frac{1}{2}\int u \times u^{\frac{1}{2}}du - \frac{1}{2}\int u^{\frac{1}{2}}du$</td>
<td>Let $u = 2x + 1$,then the integral becomes :
$\int u^{12}dx$
We need to change *dx* into *du*
$u = 2x + 1 \Rightarrow du = 2\, dx$
$\Rightarrow dx = \frac{du}{2}$ substituting this in the above integral:
$\int u^{12}dx = \int u^{12}\frac{du}{2} = \frac{1}{2}\int u^{12}du$
$= \frac{1}{2}\frac{u^{13}}{13} + C$; With $u = 2x + 1$:
$= \frac{1}{2}\frac{(2x+1)^{13}}{13} + C = \frac{(2x+1)^{13}}{26} + C$</td>
</tr>
<tr>
<td>$\frac{1}{2}\int u^{\frac{3}{2}}du - \frac{1}{2}\frac{u^{\frac{3}{2}}}{3/2} + C = \frac{1}{2}\frac{u^{\frac{5}{2}}}{5/2} - \frac{u^{\frac{3}{2}}}{3} + C$
$= \frac{u^{\frac{5}{2}}}{5} - \frac{u^{\frac{3}{2}}}{3} + C = \frac{(x^2+1)^{\frac{5}{2}}}{5} - \frac{(x^2+1)^{\frac{3}{2}}}{3} + C$</td>
</tr>
</table>