International Institute
for Technology and Management

Unit 05a: Mathematics for Business **Handout #5**

Basics V - Trigonometric Functions

Topic	Interpretation

Definitions

In fig. 3.1 :

Sine : $\sin\theta = \frac{o}{h}$

Cosine: $\cos\theta = \frac{a}{h}$

Tangent: $\tan\theta = \frac{o}{a} = \frac{\sin\theta}{\cos\theta}$

Cotangent:

$\cot\theta = \frac{a}{o} = \frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}$

Secant: $\sec\theta = \frac{h}{a} = \frac{1}{\cos\theta}$

Cosecant: $\csc\theta = \frac{h}{o} = \frac{1}{\sin\theta}$

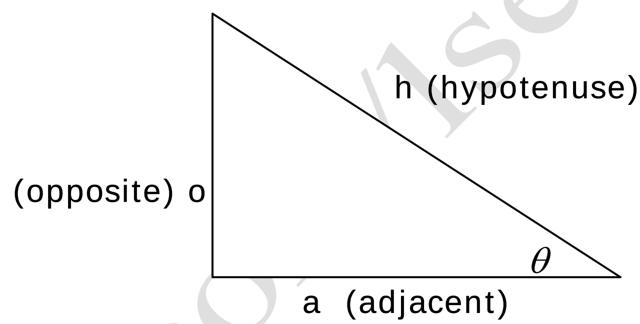


fig 3.1

Properties

$-1 \le \sin\theta \le 1$; $-1 \le \cos\theta \le 1$ so you will **never** find an angle θ such that $\cos\theta$ = 2.

$-\infty \prec \tan\theta \prec +\infty$; $-\infty \prec \cot\theta \prec +\infty$

It is acceptable to have an angle θ such that $\tan\theta$ = 200.

Measurements and Periodicity

Angles are measured either in degrees (°) or in radians (rd)

$\alpha^{\circ} = \alpha_{rd} \times \frac{180}{\pi}$; $\alpha_{rd} = \alpha^{o} \times \frac{\pi}{180}$

Periodicity:

$\sin\theta$ and $\cos\theta$ are periodic functions of period $2k\pi$

$K \in Z$ (integer)

$\cos(\alpha + 2k\pi) = \cos\alpha$;

$\sin(\alpha + 2k\pi) = \sin\alpha$

$\tan\theta$ and $\cot\theta$ are periodic functions of period $k\pi$

$K \in Z$ (integer)

$\tan(\alpha + k\pi) = \tan\alpha$;

$\cot(\alpha + k\pi) = \cot\alpha$

<u>Example1</u>: Convert $\frac{5\pi}{6} rd$ into degrees

$\frac{5\pi}{6} rd \times \frac{180}{\pi} = \frac{5 \times 180}{6} = 150°$

Convert 120° into radians:

$120° \times \frac{\pi}{180} = \frac{2\pi}{3} rd$

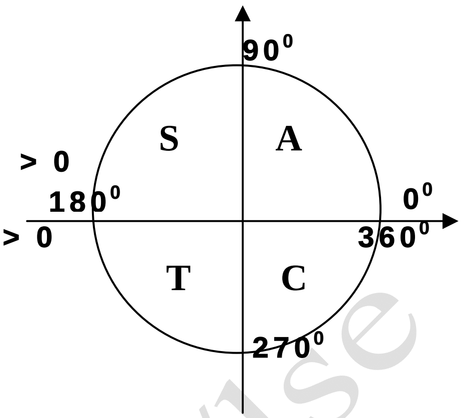
°	0	30	45	60	90	180	270	360
rd	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π

e.g. $\cos(t + 6\pi) = \cos t$; $\sin(t + 5\pi) = \sin(t + \pi + 4\pi) = \sin(t+\pi) = -\sin t$

e.g. $\tan(t + 6\pi) = \tan t$;

$\tan(t + 5\pi) = \tan t$

Sign of Trigonometric functions



<u>First Quadrant</u>: $0 \le \alpha \le \pi/2$, **ALL positive**

i.e. $\sin\alpha > 0$, $\cos\alpha > 0$, $\tan\alpha > 0$, $\cot\alpha > 0$

<u>Second Quadrant</u>: $\pi/2 \le \alpha \le \pi$, **only** $\sin\alpha > 0$

i.e. $\cos\alpha < 0$, $\tan\alpha < 0$, $\cot\alpha < 0$

<u>Third Quadrant</u>: $\pi \le \alpha \le 3\pi/2$, $\tan\alpha > 0$

Consequently $\cot\alpha > 0$, $\sin\alpha < 0$, $\cos\alpha < 0$

<u>Fourth Quadrant</u> : $3\pi/2 \le \alpha \le 2\pi$, **only** $\cos\alpha > 0$

i.e. $\sin\alpha < 0$, $\tan\alpha < 0$, $\cot\alpha < 0$

Basic Relations:

$$\csc\alpha = \frac{1}{\sin\alpha} \qquad \sec\alpha = \frac{1}{\cos\alpha} \qquad \tan\alpha = \frac{1}{\cot\alpha} = \frac{\sin\alpha}{\cos\alpha}$$

$$\sin^2\alpha + \cos^2\alpha = 1 \qquad 1 + \tan^2\alpha = \sec^2\alpha \qquad \sin\alpha = \pm\sqrt{1-\cos^2\alpha}$$

$$\cos\alpha = \pm\sqrt{1-\sin^2\alpha}$$

Double Angle Relations

$$\sin 2\alpha = 2\sin\alpha\cos\alpha = \frac{2\tan\alpha}{1+\tan^2\alpha} \qquad \tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} \qquad \cot 2\alpha = \frac{\cot^2\alpha - 1}{2\cot\alpha}$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha = \frac{1-\tan^2\alpha}{1+\tan^2\alpha}$$

Trigonometric Equations

1. $\sin x = \sin\alpha \Leftrightarrow x = \alpha + 2k\pi$ **or** $x = \pi - \alpha + 2k\pi$, $k \in Z$
2. $\cos x = \cos\alpha \Leftrightarrow x = \pm\alpha + 2k\pi$, $k \in Z$
3. $\tan x = \tan\alpha \Leftrightarrow x = \alpha + k\pi$, $k \in Z$
4. $\cot x = \cot\alpha \Leftrightarrow x = \alpha + k\pi$, $k \in Z$

Example

1. $\sin x = ½ \Leftrightarrow \sin x = \sin 30^0 = \sin\frac{\pi}{6} \Leftrightarrow x = \frac{\pi}{6} + 2k\pi$

or $x = \pi - \frac{\pi}{6} + 2k\pi = \frac{5\pi}{6} + 2k\pi$, $k \in Z$