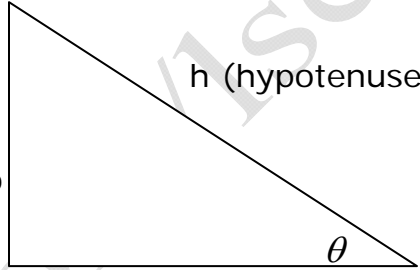


# International Institute for Technology and Management



## Unit 05a: Mathematics for Business Handout #5

### Basics V - Trigonometric Functions

Topic	Interpretation																		
<p><b>Definitions</b> In fig. 3.1 :</p> <p>Sine : <math>\sin \theta = \frac{o}{h}</math></p> <p>Cosine: <math>\cos \theta = \frac{a}{h}</math></p> <p>Tangent: <math>\tan \theta = \frac{o}{a} = \frac{\sin \theta}{\cos \theta}</math></p> <p>Cotangent:</p> <p><math>\cot \theta = \frac{a}{o} = \frac{\cos \theta}{\sin \theta} = \frac{1}{\tan \theta}</math></p> <p>Secant: <math>\sec \theta = \frac{h}{a} = \frac{1}{\cos \theta}</math></p> <p>Cosecant: <math>\csc \theta = \frac{h}{o} = \frac{1}{\sin \theta}</math></p>	 <p style="text-align: center;"><b>fig 3.1</b></p> <p><b>Properties</b>  <math>-1 \leq \sin \theta \leq 1</math> ; <math>-1 \leq \cos \theta \leq 1</math> so you will <b>never</b> find an angle <math>\theta</math> such that <math>\cos \theta = 2</math>.  <math>-\infty &lt; \tan \theta &lt; +\infty</math> ; <math>-\infty &lt; \cot \theta &lt; +\infty</math>                      It is acceptable to have an angle <math>\theta</math> such that <math>\tan \theta = 200</math>.</p>																		
<p><b>Measurements and Periodicity</b> Angles are measured either in degrees (<math>^{\circ}</math>) or in radians (rd)</p> <p><math>\alpha^{\circ} = \alpha_{rd} \times \frac{180}{\pi}</math> ; <math>\alpha_{rd} = \alpha^{\circ} \times \frac{\pi}{180}</math></p> <p><b>Periodicity:</b>  <math>\sin \theta</math> and <math>\cos \theta</math> are periodic functions of period <math>2k\pi</math>  <math>k \in \mathbb{Z}</math> (integer)  <math>\cos(\alpha + 2k\pi) = \cos \alpha</math> ;  <math>\sin(\alpha + 2k\pi) = \sin \alpha</math>  <math>\tan \theta</math> and <math>\cot \theta</math> are periodic functions of period <math>k\pi</math>  <math>k \in \mathbb{Z}</math> (integer)  <math>\tan(\alpha + k\pi) = \tan \alpha</math> ;  <math>\cot(\alpha + k\pi) = \cot \alpha</math></p>	<p><u>Example 1:</u> Convert <math>\frac{5\pi}{6}</math> rd into degrees</p> <p><math>\frac{5\pi}{6} \text{ rd} \times \frac{180}{\pi} = \frac{5 \times 180}{6} = 150^{\circ}</math></p> <p>Convert <math>120^{\circ}</math> into radians:</p> <p><math>120^{\circ} \times \frac{\pi}{180} = \frac{2\pi}{3} \text{ rd}</math></p> <table border="1" data-bbox="776 1570 1458 1705"> <tr> <td><math>^{\circ}</math></td> <td>0</td> <td>30</td> <td>45</td> <td>60</td> <td>90</td> <td>180</td> <td>270</td> <td>360</td> </tr> <tr> <td>rd</td> <td>0</td> <td><math>\frac{\pi}{6}</math></td> <td><math>\frac{\pi}{4}</math></td> <td><math>\frac{\pi}{3}</math></td> <td><math>\frac{\pi}{2}</math></td> <td><math>\pi</math></td> <td><math>\frac{3\pi}{2}</math></td> <td><math>2\pi</math></td> </tr> </table> <p>e.g. <math>\cos(t + 6\pi) = \cos t</math> ; <math>\sin(t + 5\pi) = \sin(t + \pi + 4\pi) = \sin(t + \pi) = -\sin t</math>                      e.g. <math>\tan(t + 6\pi) = \tan t</math> ;  <math>\tan(t + 5\pi) = \tan t</math></p>	$^{\circ}$	0	30	45	60	90	180	270	360	rd	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
$^{\circ}$	0	30	45	60	90	180	270	360											
rd	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$											

For comments, corrections, etc...Please contact Ahnaf Abbas: [ahnaf@uaemath.com](mailto:ahnaf@uaemath.com)

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## Sign of Trigonometric functions

**First Quadrant:**  $0 \leq \alpha \leq \pi/2$ , ALL positive

i.e.  $\sin \alpha > 0$ ,  $\cos \alpha > 0$ ,  $\tan \alpha > 0$ ,  $\cot \alpha > 0$

**Second Quadrant:**  $\pi/2 \leq \alpha \leq \pi$ , only  $\sin \alpha > 0$

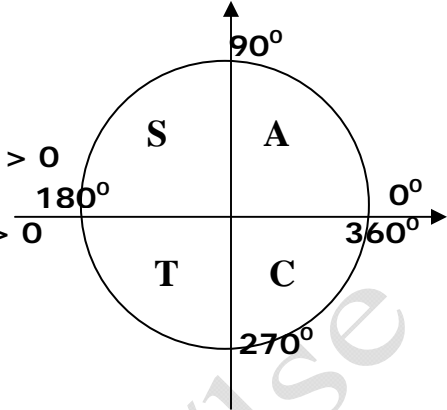
i.e.  $\cos \alpha < 0$ ,  $\tan \alpha < 0$ ,  $\cot \alpha < 0$

**Third Quadrant:**  $\pi \leq \alpha \leq 3\pi/2$ ,  $\tan \alpha > 0$

Consequently  $\cot \alpha > 0$ ,  $\sin \alpha < 0$ ,  $\cos \alpha < 0$

**Fourth Quadrant :**  $3\pi/2 \leq \alpha \leq 2\pi$ , only  $\cos \alpha > 0$

i.e.  $\sin \alpha < 0$ ,  $\tan \alpha < 0$ ,  $\cot \alpha < 0$



## Basic Relations:

$$\csc \alpha = \frac{1}{\sin \alpha} \quad \sec \alpha = \frac{1}{\cos \alpha} \quad \tan \alpha = \frac{1}{\cot \alpha} = \frac{\sin \alpha}{\cos \alpha}$$

$$\sin^2 \alpha + \cos^2 \alpha = 1 \quad 1 + \tan^2 \alpha = \sec^2 \alpha \quad \sin \alpha = \pm \sqrt{1 - \cos^2 \alpha}$$

$$\cos \alpha = \pm \sqrt{1 - \sin^2 \alpha}$$

## Double Angle Relations

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha = \frac{2 \tan \alpha}{1 + \tan^2 \alpha} \quad \tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha} \quad \cot 2\alpha = \frac{\cot^2 \alpha - 1}{2 \cot \alpha}$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = 2 \cos^2 \alpha - 1 = 1 - 2 \sin^2 \alpha = \frac{1 - \tan^2 \alpha}{1 + \tan^2 \alpha}$$

## Trigonometric Equations

1.  $\sin x = \sin \alpha \Leftrightarrow x = \alpha + 2k\pi$  or  $x = \pi - \alpha + 2k\pi$ ,  $k \in \mathbb{Z}$
2.  $\cos x = \cos \alpha \Leftrightarrow x = \pm \alpha + 2k\pi$ ,  $k \in \mathbb{Z}$
3.  $\tan x = \tan \alpha \Leftrightarrow x = \alpha + k\pi$ ,  $k \in \mathbb{Z}$
4.  $\cot x = \cot \alpha \Leftrightarrow x = \alpha + k\pi$ ,  $k \in \mathbb{Z}$

## Example

$$1. \sin x = 1/2 \Leftrightarrow \sin x = \sin 30^\circ = \sin \frac{\pi}{6} \Leftrightarrow x = \frac{\pi}{6} + 2k\pi$$

$$\text{or } x = \pi - \frac{\pi}{6} + 2k\pi = \frac{5\pi}{6} + 2k\pi, k \in \mathbb{Z}$$