International Institute
for Technology and Management

Unit 05a : Mathematics 1 **Handout #19**

Sequences & Series

Study Guide pp 120-125

Topic	Interpretation

Topic

Arithmetic Sequence(*Progression*)
$a, a+d, a+2d, a+3d, \ldots, a+(n-1)d$
n^{th} term : $a_n = a + (n-1)d$
Sum of first n terms:

$$S_n = \frac{n}{2}(a + a_n)$$
$$= \frac{n}{2}\{\ 2a + (n-1)d\ \}$$

Example 3:
Find the sum of the first n odd positive integers:
$1+3+5+7+\ldots\ldots+$

Geometric Sequence(*Progression*)
$a, ar, ar^2, ar^3, \ldots\ldots\ldots, ar^{n-1}$
n^{th} term : $a_n = ar^{n-1}$
Sum of first n terms:

$$S_n = a \times \frac{r^n - 1}{r - 1}$$

Sum to infinity of a G.P.
A geometric sequence is said to be infinite when **$-1 < r < 1$**
In this case **:**

$$S_\infty = \frac{a}{1-r}$$

Example 6:
Find the sum to infinity of :
$1+(1/2)+(1/2)^2+(1/2)^3 + \ldots\ldots\ldots$

Interpretation

Example1:
Fourth term = a+3d
15^{th} term = a + 14d
100^{th} term = a + 99d

Example2:
For the progression: 3,7,11,15,……
Find the 50^{th} term.
a = 3 ; d = 7 – 3 = 4
$a_{50} = a + 49d = 3 + 49(4) = 199$
Find the sum of the first 20 terms of the above sequence :
$S_{20} = (20/2)[2(3) + (20-1)(3)]$
$= (10)[6+57] = 630$

It is an A.P. of first term a=1 and common difference d = 2
$S = (n/2)[2(1) + (n-1)(2)]$
$= (n/2)[2n] = n^2$

Example 4:
7^{th} term = ar^6
40^{th} term = ar^{39}

Example 5:
For the sequence 3,6,12,……..
Find the 10^{th} term.
a = 3 , r = 6/3 = 2
$a_{10} = ar^9 = 3(2)^9$
Find the sum of the first 15 terms:

$$S_n = a \times \frac{r^n - 1}{r - 1} \ ; \ S_{15} = 3 \times \frac{2^{15} - 1}{2 - 1}$$

$S_{15} = 3(2^{15} - 1)$

⇨ a = 1 , r = ½ < 1
$S = a/(1-r) = 1/(1-1/2) = 2$

Compound Interest

If **P** amount is deposited in an account where the interest is paid annually at a *rate* **r** ,then after **t** *years* ,we have a balance of **P(1+r)^t**

Rate Distributed

If the year is divided over **m** *equal* periods ,the rate r/m over each period and the balance becomes **P(1+r/m)^m**

Compound interest and Exponential :

Continuous compounding :

when m tends to infinity :

As m---> ∞ , $(1+r/m)^m = e^r$

The balance after one year:

$P(1+r/m)^m = Pe^r$

For another year :$(Pe^r)(e^r) = Pe^{2r}$

After **t** years: **Pe^{rt}**

Solution of Difference Equations

(1) First order : We have difference equation

$y_k = ay_{k-1} + b$ with initial value y_0

k = 1 : $y_1 = ay_0 + b$

k = 2:

$y_2 = ay_1 + b = a[ay_0 + b] + b$

$y_2 = a^2 y_0 + b(1+a)$

k= 3 : $y_3 = ay_2 + b$

$= a[a^2 y_0 + b(1+a)] + b$

$y_3 = a^3 y_0 + b(1+a+a^2)$

k = n : $y_n = ay_{n-1} + b$

$y_n = a[a^{n-1} y_0 + b(1+a+a^2+\ldots+a^{n-2})] + b$

$y_n = a^n y_0 + b(1+a+a^2+\ldots.a^{n-1})$

$1+a+a^2+\ldots.a^{n-1}$ is a GP of first term 1 , number of terms n and common ratio r = a :

$1+a+a^2+\ldots.a^{n-1} =$

$a_1 \times (1-r^n/1-r) = 1 \times (1-a^n/1-a)$,

However if a =1 then $1+a+a^2+\ldots.a^{n-1} = n$

$y_n = y_0 + bn$ if a = 1

$y_n = a^n y_0 + b(1-a^n/1-a)$ if $a \neq 1$

<u>Examples:</u> $y_k = 2y_{k-1} - 5$; $y_0 = y_0$

$y_n = a^n y_0 + b(1-a^n/1-a)$

a = 2 ; b = - 5

$y_n = 2^n y_0 - 5(1-2^n/1-2)$

$y_n = 2^n y_0 + 5(1-2^n) =$

$= 2^n y_0 + 5 - 5\times 2^n = 5 + 2^n(y_0 - 5)$

<u>Example 7:</u>

Suppose that $1000 is invested in an account that pays interest at a fixed rate of 7% paid annually. How much is there in the account after 4 years?

$P(1+r)^t$ = $1000(1+0.07)^4$ = $1310.80

<u>Example 8:</u>

Suppose that $100 is invested in an account that pays interest at a fixed rate of 8% paid annually. How much is there in the account by the end of the year?

$P(1+r)^t = 100(1+0.08)^1$ = $ 108

*Suppose that the interest is added ***twice*** yearly. the balance : r = 2

$P(1+r/m)^m$ = $100(1+0.08/2)^2$

$=100(1+0.04)^2$ =$108.16

*Suppose that the interest is paid **quarterly** : r = 4

$P(1+r/m)^m$ = $100(1+0.08/4)^4$

$=100(1+0.02)^4$ =$108.24

<u>Example 9:</u>

Suppose $ 5000 is invested at an annual rate of 4% ***compounded continuously*** for 5 years. Find the compound amount.

P = 5000 , r = 0.04 , t = 5

$Pe^{rt} = 5000e^{(0.04)(5)} = 5000e^{0.02}$

= $ 6107.01