International Institute for
Technology and Management

Unit 05a : Mathematics 1

Handout #11

Integration III: *Anthony & Biggs pp: 333 – 341*

Topic	Interpretation
Integration by parts $\int u\,dv = uv - \int v\,du$ Example: $\int x e^x\,dx$	Let u = x ⇒ du = dx and dv = e^x dx ⇒ v = $\int e^x dx = e^x$ $\int x e^x\,dx = uv - \int v\,du = xe^x - \int e^x\,dx$ $= xe^x - e^x + C$
Integration by partial Fractions *Reminder:* $\int \frac{dx}{ax+b} = \frac{1}{a}\ln(ax+b)$ Example: $\int \frac{dx}{x-2} = \ln(x-2)$ $\int \frac{p(x)}{q(x)}\,dx$ degree p(x) < degree q(x) $= \frac{a}{x-x_1} + \frac{b}{x-x_2} + \ldots$ Example: $\int \frac{2x+1}{x^2+5x+6}\,dx$	$\frac{2x+1}{x^2+5x+6} = \frac{a}{x+2} + \frac{b}{x+3}$ Multiplying by $x^2 + 5x + 6 = (x+2)(x+3)$: 2x + 1 = a(x +3) + b(x +2) Choose x = - 2 2(-2) + 1 = a(1) +b(0) ⇒ a = -3 Choose x = - 3 2(-3) + 1 = a(0) + b(-1) ⇒ b = 5 $\frac{2x+1}{x^2+5x+6} = \frac{-3}{x+2} + \frac{5}{x+3}$ $\int \frac{2x+1}{x^2+5x+6}\,dx = \int \frac{-3}{x+2}\,dx + \int \frac{5}{x+3}\,dx$ $= -3\ln(x+2) + 5\ln(x+3) + C$
If degree p(x) ≥ degree q(x)	Try Long division.

Example: $\int \frac{t^2+1}{3t+t^3}\,dt$

Although degree of t^2+1 is < degree $3t + t^3$; No need for partial fractions

Note that if $u = 3t + t^3 \Rightarrow du = (3 + 3t^2)dt = 3(1 + t^2)dt$

$\Rightarrow \frac{du}{3} = (1+t^2)dt$ substituting in the integral :

$$\int \frac{t^2+1}{3t+t^3}\,dt = \int \frac{\frac{du}{3}}{u} = \frac{1}{3}\int \frac{du}{u} = \frac{1}{3}\ln u + C = \frac{1}{3}\ln(3t+t^3)+C$$

Example: Show that $\frac{x^3+2}{x^2-1} = x + \frac{x+2}{x^2-1}$

Using this result and the method of partial fractions ,determine :

$$\int \frac{x^3+2}{x^2-1}\,\mathrm{dx}$$

$$x + \frac{x+2}{x^2-1} = \frac{x^3-x+x+2}{x^2-1} = \frac{x^3+2}{x^2-1}$$

$$\int \frac{x^3+2}{x^2-1}\,\mathrm{dx} = \int x\,\mathrm{dx} + \int \frac{x+2}{x^2-1}\,\mathrm{dx}$$

The first one : $\int x\,\mathrm{dx} = \frac{x^2}{2}$; The second one by partial fractions:

$\frac{x+2}{x^2-1} = \frac{a}{x-1} + \frac{b}{x+1}$; Multiplying by x2-1 = (x-1)(x+1)

x+2 = a(x+1) + b(x-1)

Choose x = 1 : 1 + 2 = a(2) +b(0) $\Rightarrow$ a = 3/2

Choose x = - 1 : -1 + 2 = a(0) + b(-2) $\Rightarrow$ b = -1/2

$$\frac{x+2}{x^2-1} = \frac{\frac{3}{2}}{x-1} + \frac{\frac{-1}{2}}{x+1} \Rightarrow \int \frac{x+2}{x^2-1}\,\mathrm{dx} = \frac{3}{2}\int \frac{dx}{x-1} - \frac{1}{2}\int \frac{dx}{x+1}$$

$$= \frac{3}{2}\ln(x-1) - \frac{1}{2}\ln(x+1)$$

$$\int \frac{x^3+2}{x^2-1}\,\mathrm{dx} = \int x\,\mathrm{dx} + \int \frac{x+2}{x^2-1}\,\mathrm{dx} = \frac{x^2}{2} + \frac{3}{2}\ln(x-1) - \frac{1}{2}\ln(x+1) + C$$