International Institute for Technology and Management

February 16, 2009 **Unit: 05a – Mathematics 1**

GROUP: A Duration: 90 minutes

Answer all of the following questions:

1. (a) Suppose that **f(x,y) = $x^2y + y^2$** .
Let **x = $3t^2 + 3$** and y = **$t^3 - 7$** , Use the chain rule to find **F'(2).**

(10 Marks)

(b) If $x^{\sqrt{y}} = 7$ **find** $\frac{dy}{dx}$

(10 Marks)

2. (a) Find and classify the stationary points of the function

f(x,y) = $x^2 - 2x - y^3 + y^2 + 8$

(10 Marks)

(b) The function $f(x, y)$ is defined for $x, y > 0$ by

$$f(x,y) = \frac{ye^{2y}}{x^a},$$

where a is a fixed real number Find expressions for the partial derivatives

$$\frac{\partial f}{\partial x}, \quad \frac{\partial f}{\partial y}, \quad \frac{\partial^2 f}{\partial x^2}, \quad \frac{\partial^2 f}{\partial y^2}$$

Determine the values of a for which the function will satisfy the equation

$$yx^2\frac{\partial^2 f}{\partial x^2} - 3y\frac{\partial^2 f}{\partial y^2} + 12f = 0.$$

(15 Marks)

3. Use the Lagrange multiplier method to find the maximum value of

$$\left(\frac{1}{x^2} + \frac{1}{y^2}\right)^{-1/2}$$

among all positive x, y satisfying $x + y = \sqrt{2}$

(10 Marks)

4. If

$$f(x,y) = y\ln\left(\frac{y}{x}\right) + xe^{x/y}$$

(defined for positive x and y), find the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ and show that

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = f.$$

(15 Marks)

5. (a) A firm is the only producer of two goods, X and Y. The demand equations for X and Y are given by

$$x = 50 - \frac{1}{2}p_X, \quad y = 240 - 2p_Y,$$

where x and y are the quantities of X and Y demanded (respectively) and p_X, p_Y are (respectively) the prices of X and Y. The firm's joint total cost function (that is, the cost of producing x of X and y of Y) is

$$x^2 + 2xy + y^2 + 10.$$

Find an expression in terms of x and y for the profit function. Determine the quantities x and y that maximise the profit.

(15 Marks)

(b) A consumer has utility function

$$u(x,y) = 3\ln x_1 + \ln x_2$$

for two goods, X_1 and X_2. (Here, x_1 and x_2 are, respectively, the amounts of X_1 and X_2 consumed.) Suppose that each unit of X_1 costs $\$p_1$ and each unit of X_2 costs $\$p_2$, and that the consumer has a budget of M to spend on these two goods. By using the Lagrange multiplier method, determine the quantities x_1^* and x_2^* of X_1 and X_2 that maximise the consumer's utility function subject to the constraint on his budget.

(15 Marks)

END OF PAPER