International Institute for
Technology and Management
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Unit 05a: Mathematics 1 – (MathB) Assignment – 1

1. The supply equation for a good is **q = 2p² - 38p + 39** $q = 2p^2 - 38p + 39$
and the demand equation is $q = 48 - 2p - p^2$
Sketch the supply and the demand functions for $p \geq 0$
Determine the equilibrium price and quantity.

The Supply $q = 2p^2 - 38p + 39$

(1)It has U shape since it has positive p^2 term

(2)Intercepts: p-intercepts : q = 0 $\Rightarrow 2p^2 - 38p + 39 = 0$

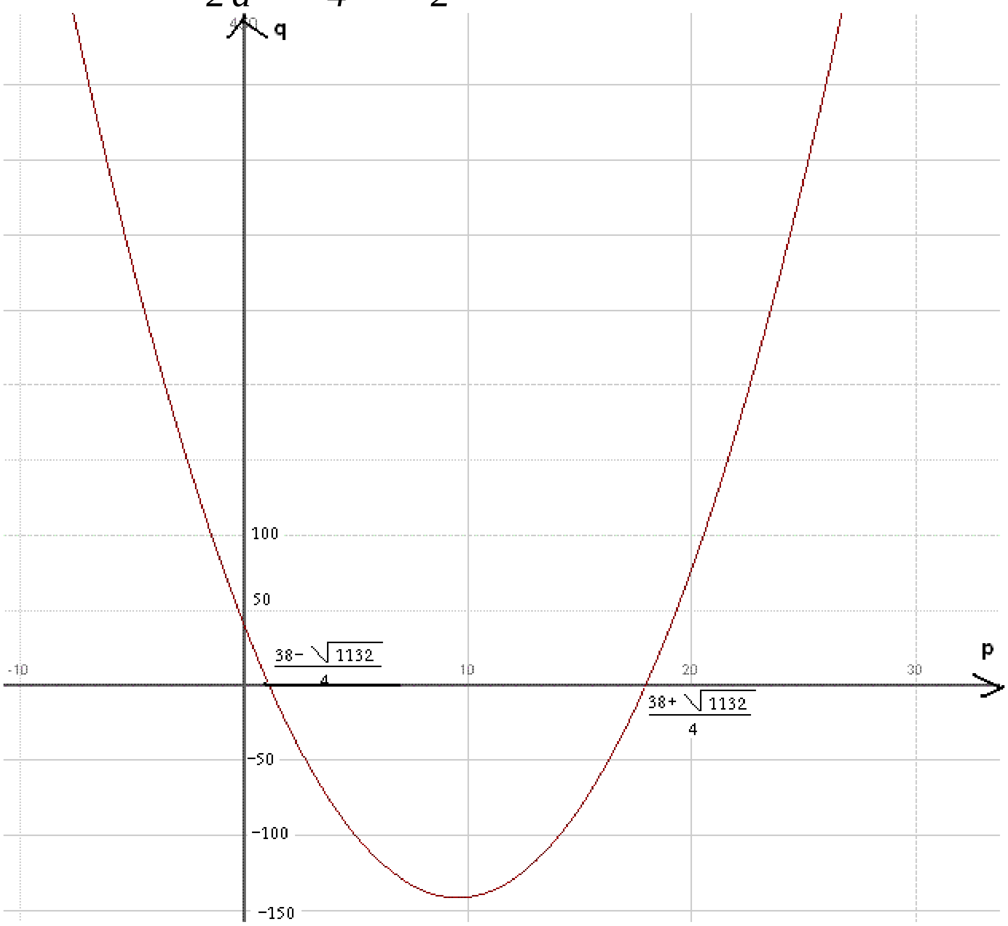
$$p = \frac{38 \pm \sqrt{38^2 - 4(2)(39)}}{4} = \frac{38 \pm \sqrt{1444 - 312}}{4} = \frac{38 \pm \sqrt{1132}}{4}$$

q-intercpt: p = 0 $\Rightarrow$ q =39 ; (0,39)

(3) The minimum : q′ = 4p - 38= 0 $\Rightarrow$ p =19/2

$\Rightarrow$ $q = 2(19/2)^2 - 38(19/2) + 39 = -283/2$; V(19/2 , -283/2)

OR $p = \frac{-b}{2a} = \frac{38}{4} = \frac{19}{2} \Rightarrow q = -283/2$



The demand : **q = - p² - 2p + 48**

(1) It has $\cap$ shape since it has negative p^2 term .

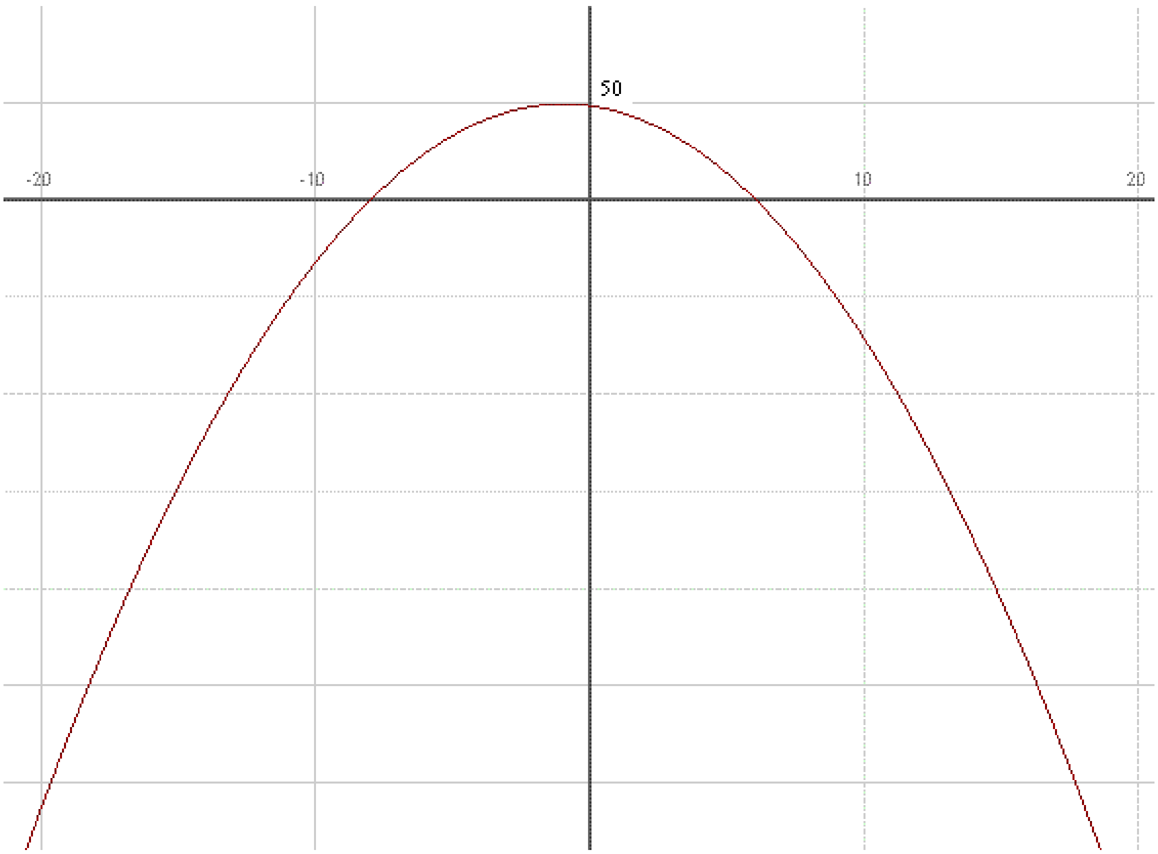
(2) Intercepts: <u>p-intercept</u> , q = 0 $\Rightarrow$ **- p^2 - 2p + 48**

$$p = \frac{1 \pm \sqrt{4^2 - 4(-1)(48)}}{-2} = \frac{1 \pm \sqrt{192}}{-2} = \frac{-(1 \pm \sqrt{192})}{2}$$

<u>q-intercepts</u> : p = 0 $\Rightarrow$ q = 48 ; (0 , 48)

(3) The maximum : q′ = -2p -2 = 0 $\Rightarrow$ p=-1 $\Rightarrow$q =51

OR p = $\frac{-b}{2a} = \frac{2}{-2}$ =-1$\Rightarrow$ q =51 $\Rightarrow$ V(-1,51)



Equilibrium price and quantity : q = q

$\Rightarrow$ **$2p^2$ - 38p + 39 = 48 - 2p - p^2** $\Rightarrow$ **3 p^2 – 36p – 9 = 0**

$\Rightarrow$ **p^2 – 12 p – 3 = 0**

$$p = \frac{12 \pm \sqrt{12^2 - 4(1)(-3)}}{2} = \frac{12 \pm \sqrt{144 + 12}}{2} = \frac{12 \pm \sqrt{156}}{2} = \frac{12 \pm \sqrt{4 \times 39}}{2}$$

$p = \frac{12 \pm 2\sqrt{39}}{2} = 6 \pm \sqrt{39} \Rightarrow p = 6 + \sqrt{39}$ but **q = - p^2 - 2p + 48**

q = - $(6 + \sqrt{39})^2 - 2(6 + \sqrt{39}) + 48 = -(36 + 12\sqrt{39} + 39) - 12 - \sqrt{39} + 48$

q = 39 + 11$\sqrt{39}$

2. A monopolist's average cost function is given by :

$$2+3q-\frac{5}{q}$$

Where q is the quantity produced, the demand function for the good is $q = 10 - \frac{p}{2}$

Determine expressions, in terms of q , for the revenue and The profit and determine the value of q that maximizes the profit. Find the maximum profit.

Revenue = Demand × Price = p × q

$q = 10 - \frac{p}{2} \Rightarrow p = -2q+20$

$TR = q \times (-2q+20) = -2q^2+20q$

Profit = Revenue – Cost

$AC = 2+3q-\frac{5}{q} \Rightarrow TC = q \times AC = 2q+3q^2-5$

Profit: $\Pi = TR - TC = -2q^2+20q - (2q+3q^2-5)$

$\Pi = -5q^2+18q+5$

q=? so that ∏ is maximum : Vertex abscissa **x =** $\frac{-b}{2a} = \frac{9}{5}$

or $\frac{d\Pi}{dq} = 0 \Rightarrow -10q+18 = 0 \Rightarrow q = \frac{18}{10} = \frac{9}{5}$

Maximum profit ?

$$\Pi = -5q^2+18q+5 = -5\left(\frac{9}{5}\right)^2+18\left(\frac{9}{5}\right)+5 = \frac{106}{5}$$

3. Solve each of the following equations/inequalities:

1. $-x^4 + 10x^2 - 9 = 0$
   a+b+c = 0 $\Rightarrow x^2 = 1$ or $x^2 = c/a = 9$
   $\Rightarrow x = \pm 1$ or $x = \pm 3$
2. $8x^3 - 27 = 0 \Rightarrow x^3 = 27/8 \Rightarrow x = 3/2$
3. $\sqrt{2x-1} = 2-3x \Rightarrow$ **$2x - 1 = (2-3x)^2 \Rightarrow 2x-1 = 4 - 12x + 9x^2$**
   $\Rightarrow 9x^2 - 14x + 5 = 0$
   a+b+c = 0 $\Rightarrow$ x = 1 or x = c/a = 5/9

4. $\begin{cases} -\frac{3}{4}x + 8y - 37 = 0 \Rightarrow -3x + 32y - 148 = 0 \\ -35 + 8x + \frac{3}{5}y = 0 \Rightarrow 40x + 3y - 175 = 0 \end{cases}$

the first one gives: **x =** $\frac{32y - 148}{3}$ **;** substitute this in the second

40 ($\frac{32y-148}{3}$**) + 3y – 175 = 0** $\Rightarrow$ 40 (32y – 148) + 9y – 525 = 0

1280 y – 5920 + 9y – 535 = 0 $\Rightarrow$ 1289y = 6445 $\Rightarrow$ y = $\frac{6445}{1289} = 5$

But **x =** $\frac{32y-148}{3}$ **=** $\frac{32(5)-148}{3} = \frac{12}{3} = 4 \Rightarrow$ **(x,y)= (4,5)**

5. **|7x -5 | -1 > 10** $\Rightarrow$ **|7x -5 | > 11**
$\Rightarrow$ **7x -5 < - 11 or 7x -5 > 11**
$\Rightarrow$ **x < - 6/7 or x > 16/7**

6. **|8x+1| -13 < 4** $\Rightarrow$ **|8x+1| < 17** $\Rightarrow$ **-17 < 8x+1< 17**
$\Rightarrow$ **-18 < 8x< 16** $\Rightarrow$ **-18/8 < x < 2** $\Rightarrow$ **-9/4 < x < 2**

7. $e^x + 3e^{-x} = 4 \Rightarrow$ $\mathbf{e^x + 3/e^x = 4}$ $\Rightarrow$ $\mathbf{e^{2x} - 4e^x + 3 = 0}$
a+b+c = 0 $\Rightarrow$ $\mathbf{e^x = 1}$ $\Rightarrow$ x = ln1 = 0 **or** $\mathbf{e^x = c/a = 3}$ $\Rightarrow$ x=ln3

8. ln(3x+2) = ln4 – ln(x+2) $\Rightarrow$ ln(3x+2) + ln(x+2)= ln4
$\Rightarrow$ ln(3x+2)(x+2) = ln4 $\Rightarrow$ (3x+2)(x+2) = 4 $\Rightarrow$ $3x^2 + 8x = 0$
$\Rightarrow$ x(3x + 8) = 0 $\Rightarrow$ x = 0 or x = -8/3

9. $(\ln x)^2 + \ln x^2 - 1 = 0 \Rightarrow (\ln x)^2 + 2\ln x - 1 = 0$

$$\ln x = \frac{-2 \pm \sqrt{4 - 4(1)(-1)}}{2} = \frac{-2 \pm \sqrt{8}}{2} = \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$$

ln x = $-1 - \sqrt{2} \Rightarrow$ **x =** $e^{-1-\sqrt{2}}$ **or ln x =** $-1 + \sqrt{2} \Rightarrow$ **x =** $e^{-1+\sqrt{2}}$

10. Solve the system :
ln x + lny = 0 , x + y = 2
lnxy = 0 $\Rightarrow$ **xy =** $\mathbf{e^0}$ **= 1** $\Rightarrow$ **y =** $\frac{1}{x}$
x +y = 2 $\Rightarrow$ **x +** $\frac{1}{x}$ **= 2** $\Rightarrow$ $\mathbf{x^2 - 2x + 1 = 0}$
$\Rightarrow$ $\mathbf{(x-1)^2 = 0}$ $\Rightarrow$ **x = 1**

4. Given that a company has a linear cost function and that it costs \$ 600 to produce 4 units and \$ 700 to produce 8 units.Determine the cost C(x) of producing x units.

Linear cost function : C = aq + b

For q = 4 , C = 620 $\Rightarrow$ 600 = 4a + b -------(1)

For q = 8 , C = 700 $\Rightarrow$ 700 = 8a + b -------(2)

Solving simultaneously , by subtracting (1) from (2):

4a = 100 $\Rightarrow$ a = 25 ,

using (1) : b = 600 – 4a = 600 – 100 = 500

$\Rightarrow$ C = 25q + 500

5. A computer manufacturer finds that when x millions of dollars are spent on research, the profit, P(x), in millions of dollars, is given by $P(x) = 20 + 5\log_3(x+3)$. How much should be spent on research to make a profit of 40 million dollars?

$P(x) = 20 + 5\log_3(x+3)$ = 40

$$5\log_3(x+3) = 20 \Rightarrow \log_3(x+3) = 4 \Rightarrow x+3 = 3^4$$

x= 81 – 3 = 78

6.

The inverse supply and demand functions for a market are given by the equations

$$p^S(q) = 2q + 3 \qquad \text{and} \qquad p^D(q) = -q^2 - 2q + 8,$$

respectively.

(a) Write $p^D(q)$ in completed square form and determine the coordinates and nature of the turning point of the curve $p = p^D(q)$.

(b) Determine the p and q-intercepts of the curves $p = p^S(q)$ and $p = p^D(q)$.

(c) Find the points of intersection of the curves $p = p^S(q)$ and $p = p^D(q)$. Hence, deduce the equilibrium price and quantity for this market.

(d) Sketch both of these curves on the same axes clearly indicating which parts of these curves are economically meaningful.

(a) $p = -q^2 - 2q + 8 = -q^2 - 2q - 1 + 9 = -(q^2 + 2q + 1) + 9$

$p = -(q+1)^2 + 9$

turning point is the vertex : q = -b/2a = - 1 substitute this in the equation : p = 9 , vertex is V(-1,9)

(b) Intercepts of the supply curve :

p-intercept : $q = 0 \Rightarrow p = 3$ (0 ,3)

q- intercept: $p = 0 \Rightarrow q = -3/2$ (-3/2,0)

Intercepts of the Demand curve:

p-intercept : $q = 0 \Rightarrow p = 10$ (0 ,10)

q- intercept: $p = 0 \Rightarrow -(q+1)^2 + 9 \Rightarrow (q+1)^2 = 9$

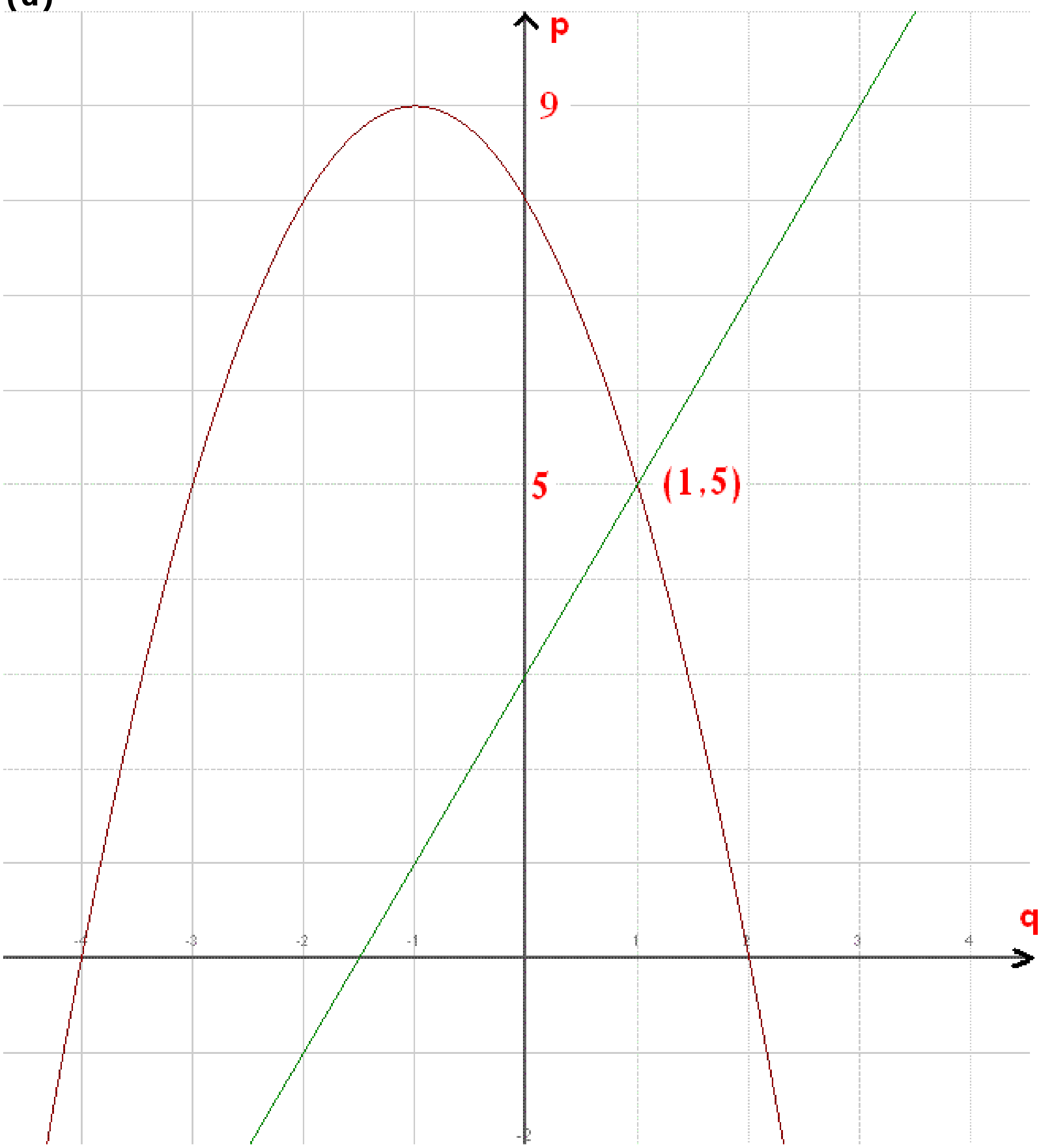
$\Rightarrow q+1 = \pm 3$ $\Rightarrow$ q = 2 or q = -4 : (2,0) ; (-4,0)

(c) $p = p \Rightarrow -q^2 - 2q + 8 = 2q + 3 \Rightarrow -q^2 - 4q + 5 = 0$

$a+b+c = 0 \Rightarrow q = 1$ or $q = c/a = -5$ rejected

i.e. $q = 1 \Rightarrow p = 2q + 3 = 2(1) + 3 = 5$

(d)



6. A firm's total costs are TC = $\frac{1}{3}q^3 - 5q^2 + 30q$

(i) Determine the firm's average cost (AC) function.

(ii) Find the value of q that makes the firm's average cost minimum and find this minimum.

(iii) Assume this firm operates in a perfectly competitive market and is able to sell its output at a price of £14 per unit. Determine its profit function.

(i) AC = $\frac{TC}{q} = \frac{1}{3}q^2 - 5q + 30$

(ii) Find the value of q that makes the firm's average cost minimum. Verify that it is a minimum and find this minimum.

$$\frac{d}{dq}AC = \frac{2}{3}q - 5 = 0 \Rightarrow q = \frac{15}{2}$$

$$\frac{d^2}{dq^2}AC = \frac{2}{3} > 0 \Rightarrow q = \frac{15}{2}$$ Minimises AC

Minimum = $AC(\frac{15}{2}) = \frac{135}{12}$

(iii) Assume this firm operates in a perfectly competitive market and is able to sell its output at a price of £14 per unit. Determine its profit-maximising level of output.

TR = 14 q , TC = $\frac{1}{3}q^3 - 5q^2 + 30q$

π = TR – TC = $14q - (\frac{1}{3}q^3 - 5q^2 + 30q)$

$\pi = -\frac{1}{3}q^3 + 5q^2 - 16q$; $\frac{d\pi}{dq} = -q^2 + 10q - 16 = 0$

(q -2)(q-8) = 0 $\Rightarrow$ Either q = 2 or q = 8

$\frac{d^2\pi}{dq^2} = -2q + 10$; $\frac{d^2\pi}{dq^2}(8) = -2(8) + 10 = -6 < 0$

Hence q = 8 maximises the profit.

With q = 2(minimizes) the second derivative = 6 > 0

END of QUESTIONS