International Institute for
Technology and Management
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Unit 05a: Mathematics 1 – (MathA) Assignment – 1

1. (a) The supply equation for a good is

$$q = 10p^2 + 2p$$

and the demand equation is

$$q = 150 - 6p^2$$

where **p** is the price.
Sketch the supply and the demand functions for **p** ≥ 0
Use the graph , or otherwise , to find the positive **p** at which the two curves intersect.

The Supply $q = 10p^2 + 2p$

(1)It has U shape since it has positive p^2 term

(2)Intercepts: <u>p-intercepts</u> : q = 0 $\Rightarrow 10p^2 + 2p = 0$

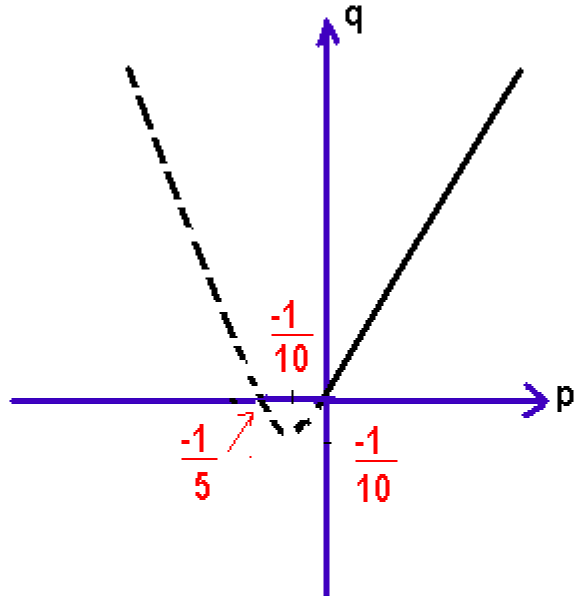
$2p(5p+1)=0 \Rightarrow p=0$ or $p=-1/5$; (0,0) and (-1/5,0)

<u>q-intercpt</u>: p = 0 $\Rightarrow$ q =0 ; (0,0)

(3) The minimum : $q' = 20p + 2 = 0 \Rightarrow p = -1/10$

$\Rightarrow q = -1/10$; (-1/10 , -1/10)

OR $p = \frac{-b}{2a} = \frac{-2}{20} = \frac{-1}{10} \Rightarrow q = -1/10 \Rightarrow V(-1/10,-1/10)$



The demand : $q = 150 - 6p^2$

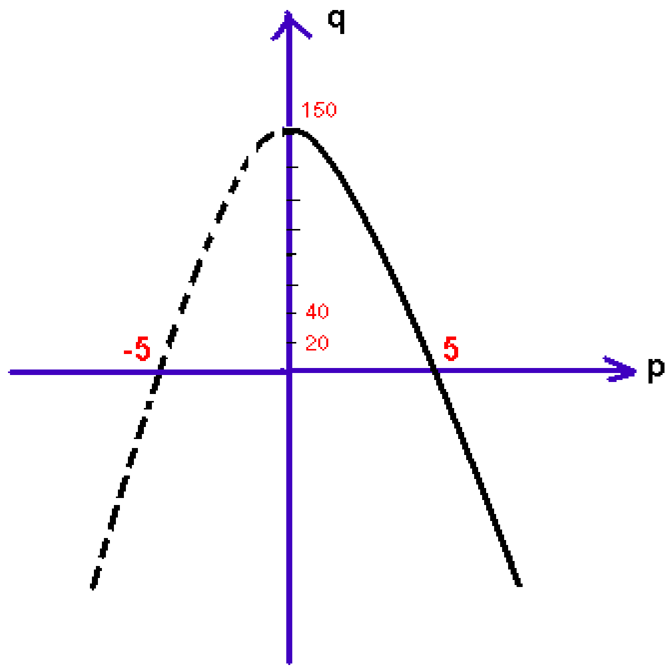
(1) It has ∩ shape since it has negative p^2 term .

(2) Intercepts: <u>p-intercept</u> , q = 0 $\Rightarrow 150 - 6p^2 = 0$

$p^2 = 25 \Rightarrow p = -5$ or $p = 5$;(-5,0) and (5,0) q-intercepts : $p = 0 \Rightarrow q = 150$; (0 , 150)

(3) The maximum : $q' = -12p = 0 \Rightarrow p=0 \Rightarrow q = 150$

OR $p = \frac{-b}{2a} = \frac{0}{-12} = 0 \Rightarrow q = 150 \Rightarrow V(0,150)$



To determine the equilibrium price, we solve:

$10p^2 + 2p = 150 - 6p^2 \Rightarrow 16p^2 + 2p - 150 = 0$

which is equivalent to $8p^2 + p - 75 = 0 \Rightarrow (8p+25)(p-3)=0$

Either p = -25/8 or p=3 of which only p=3 is economically Meaningful. $p=3 \Rightarrow q = 96$

(b) For which values of $\alpha \in \Re$ has the equation:

$$x^2 + x + \alpha = 0$$

No solutions, exactly one solution or two solutions?
Determine the solutions in the second and third cases.

$\mathbf{b^2 - 4ac = 1 - 4\alpha}$

No solution: $\mathbf{1 - 4\alpha < 0 \Rightarrow 4\alpha > 1 \Rightarrow \alpha > 1/4}$

Exactly one solution: $\mathbf{1 - 4\alpha = 0 \Rightarrow 4\alpha = 1 \Rightarrow \alpha = 1/4}$

Two solutions: $\mathbf{1 - 4\alpha > 0 \Rightarrow 4\alpha < 1 \Rightarrow \alpha < 1/4}$

In case of two solutions, the roots are $x = \frac{-1 \pm \sqrt{1-4\alpha}}{2}$

In case of one solution, the root is $x = \frac{-1 \pm 0}{2} = \frac{-1}{2}$

2. Solve the following equations in the set of real numbers :

a. $\frac{-5}{7}q+\frac{5}{3}q^2-\frac{20}{21}=0$ multiply the whole equation by **21**

- 15q + 35q² – 20 = 0 $\Rightarrow$ **35q² - 15q – 20 = 0**

a+b+c = 0 $\Rightarrow$ q = 1 or q = c/a = -20/35 = -4/7

b. $\begin{cases} -\frac{3}{4}x+8y-37=0 \Rightarrow -3x+32y-148=0 \\ -35+8x+\frac{3}{5}y=0 \Rightarrow 40x+3y-175=0 \end{cases}$

the first one gives: **x =** $\frac{32y-148}{3}$ **;** substitute this in the second

40 ($\frac{32y-148}{3}$**) + 3y – 175 = 0** $\Rightarrow$ 40 (32y – 148) + 9y – 525 = 0

1280 y – 5920 + 9y – 535 = 0 $\Rightarrow$ 1289y = 6445 $\Rightarrow$ y = $\frac{6445}{1289}=5$

But **x =** $\frac{32y-148}{3}$ **=** $\frac{32(5)-148}{3}=\frac{12}{3}=4 \Rightarrow$ **(x,y)= (4,5)**

c. $(\ln x)^2+\ln x^2-1=0 \quad \Rightarrow \quad (\ln x)^2+2\ln x-1=0$

$$\ln x=\frac{-2\pm\sqrt{4-4(1)(-1)}}{2}=\frac{-2\pm\sqrt{8}}{2}=\frac{-2\pm2\sqrt{2}}{2}=-1\pm\sqrt{2}$$

ln x = $-1-\sqrt{2} \Rightarrow$ **x =** $e^{-1-\sqrt{2}}$ **or ln x =** $-1+\sqrt{2} \Rightarrow$ **x =** $e^{-1+\sqrt{2}}$

d. $2e^{x^2}+2x(2x-3)e^{x^2}=0 \Rightarrow 2e^{x^2}+(4x^2-6x)e^{x^2}=0$

$\Rightarrow e^{x^2}(2+4x^2-6x)=0$ but $e^{x^2}\neq 0 \Rightarrow 4x^2-6x+2$ **= 0**

a+b+c = 0 $\Rightarrow$ **x = 1 or x = c / a = 2 / 4 = ½**

e. ln x + lny = 0 $\Rightarrow$ **lnxy = 0** $\Rightarrow$ **xy = e⁰ = 1** $\Rightarrow$ **y =** $\frac{1}{x}$

x +y = 2 $\Rightarrow$ **x +** $\frac{1}{x}$ **= 2** $\Rightarrow$ **x² – 2x + 1 = 0** $\Rightarrow$ **(x-1)²=0**$\Rightarrow$ **x = 1**

y = 2 – x = 2 – 1 = 1 $\Rightarrow$ **(x , y)= (1 , 1)**

f. $|7x - 5| - 1 > 10 \Rightarrow |7x - 5| > 11$
$\Rightarrow 7x - 5 < -11$ or $7x - 5 > 11$
$\Rightarrow x < -6/7$ or $x > 16/7$

g. $|8x+1| - 13 < 4 \Rightarrow |8x+1| < 17 \Rightarrow -17 < 8x+1 < 17$
$\Rightarrow -18 < 8x < 16 \Rightarrow -18/8 < x < 2 \Rightarrow -9/4 < x < 2$

h. $|x^2 - 4x + 1| = 4 \Rightarrow x^2 - 4x + 1 = \pm 4$
Either $x^2 - 4x + 1 = -4 \Rightarrow x^2 - 4x + 5 = 0 \Rightarrow$ **No roots**
Or $x^2 - 4x + 1 = 4 \Rightarrow x^2 - 4x - 3 = 0 \Rightarrow x = 2 \pm \sqrt{7}$

i. $5x - \frac{1}{x} = 4 \Rightarrow 5x^2 - 1 = 4x \Rightarrow 5x^2 - 4x - 1 = 0$
$a+b+c = 0 \Rightarrow x = 1$ or $x = c/a = -1/5$

j. $\sqrt{2x-1} = 2 - 3x \Rightarrow 2x - 1 = (2-3x)^2 \Rightarrow 2x - 1 = 4 - 12x + 9x^2$
$\Rightarrow 9x^2 - 14x + 5 = 0$
$a+b+c = 0 \Rightarrow x = 1$ or $x = c/a = 5/9$
Accepted solutions when : $2x - 1 \geq 0 \Rightarrow x \geq ½$
Therefore both are accepted being greater than ½ .

3. The demand for a commodity is given by : $p(q + 1) = 231$
and the supply is given by : $p - q = 11$. Determine the equilibrium price and quantity.

Equilibrium price and quantity : $p(q + 1) = 231 \Rightarrow p = \frac{231}{q+1}$

$p - q = 11 \Rightarrow p = 11 + q$

$p = p \Rightarrow \frac{231}{q+1} = 11 + q \Rightarrow q^2 + 12q - 220 = 0 \Rightarrow (q + 22)(q - 10) = 0$
since q can not be negative, $q = 10$. Hence $p = 11 + q = 21$

4. A monopolist's average cost function is given by : $2 + 3q - \frac{5}{q}$

Where **q** is the quantity produced, the demand function for the good is $q = 10 - \frac{p}{2}$

Determine expressions, in terms of **q** , for the revenue and the profit and determine the value of **q** that maximizes the profit. Find the maximum profit.

Revenue = Demand × Price = p × q

q = 10 - $\frac{p}{2} \Rightarrow \; p = -2q + 20$

TR = q× $(-2q+20) = -2q^2 + 20q$

Profit = Revenue – Cost

AC = $2 + 3q - \frac{5}{q}$ ⇒ TC = q ×AC = $2q + 3q^2 - 5$

Profit: ∏ =TR-TC= $-2q^2 + 20q$ - $(2q + 3q^2 - 5)$

∏ = $-5q^2 + 18q + 5$

q=? so that ∏ is maximum : Vertex abscissa **q =** $\frac{-b}{2a} = \frac{9}{5}$

or $\frac{d\prod}{dq} = 0 \Rightarrow -10q + 18 = 0 \Rightarrow$ q = $\frac{18}{10} = \frac{9}{5}$

Maximum profit ?

∏ = $-5q^2 + 18q + 5$ = $-5\left(\frac{9}{5}\right)^2 + 18\left(\frac{9}{5}\right) + 5$ = $\frac{106}{5}$

5. (20 Marks)

The inverse supply and demand functions for a market are given by the equations

$$p^S(q) = 2q + 3 \qquad \text{and} \qquad p^D(q) = -q^2 - 2q + 8,$$

respectively.

(a) Write $p^D(q)$ in completed square form and determine the coordinates and nature of the turning point of the curve $p = p^D(q)$.

(b) Determine the p and q-intercepts of the curves $p = p^S(q)$ and $p = p^D(q)$.

(c) Find the points of intersection of the curves $p = p^S(q)$ and $p = p^D(q)$. Hence, deduce the equilibrium price and quantity for this market.

(d) Sketch both of these curves on the same axes clearly indicating which parts of these curves are economically meaningful.

(a) p = $-q^2$ -2q + 8 = $-q^2$ -2q -1+ 9 = - (q^2 +2q +1) + 9
p = - $(q+1)^2$ + 9
turning point is the vertex : q = -b/2a = - 1 substitute this in the equation : p = 9 , vertex is V(-1,9)

(b) Intercepts of the supply curve :
p-intercept : q = 0 ⇒ p = 3 (0 ,3)
q- intercept: p = 0 ⇒ q = -3/2 (-3/2,0)

Intercepts of the Demand curve:

p-intercept : $q = 0 \Rightarrow p = 10$ (0 ,10)

q- intercept: $p = 0 \Rightarrow -(q+1)^2 + 9 \Rightarrow (q+1)^2 = 9$

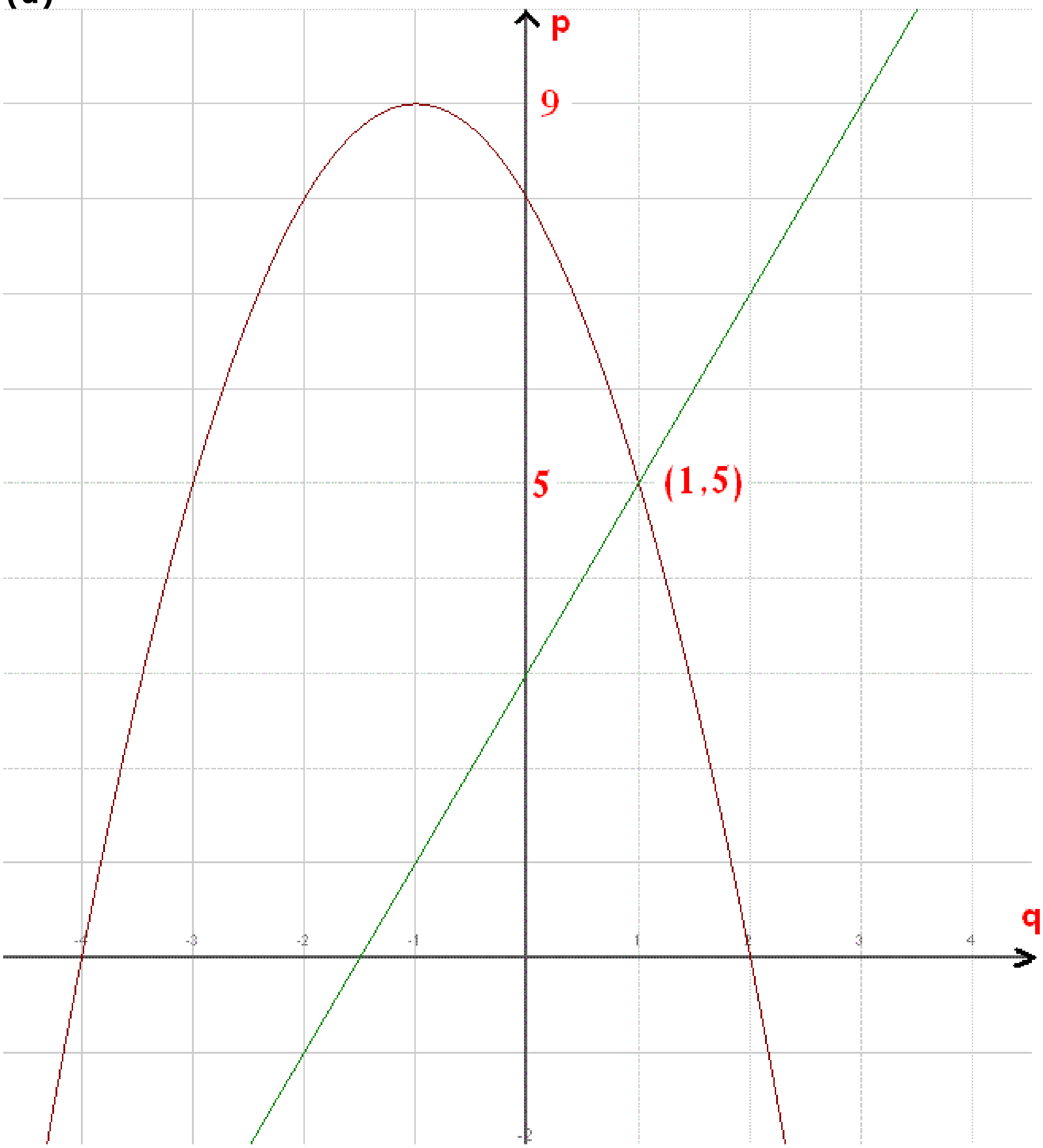
$\Rightarrow q+1 = \pm 3$ $\Rightarrow q = 2$ or $q = -4$: (2,0) ; (-4,0)

(c) $p = p \Rightarrow -q^2 - 2q + 8 = 2q + 3 \Rightarrow -q^2 - 4q + 5 = 0$

$a+b+c = 0 \Rightarrow q = 1$ or $q = c/a = -5$ rejected

i.e. $q = 1 \Rightarrow p = 2q + 3 = 2(1) + 3 = 5$

(d)



6.

A company has a profit function given by,

$$\pi(x) = -x^2 + 20x + 312,$$

where x denotes the quantity produced.

(a) Write the function $\pi(x)$ in completed square form.

(b) Find the x-intercepts and y-intercepts of the curve $y = \pi(x)$.

(c) Find the value of x that gives the maximum profit. What is the maximum profit?

(d) Use the above information to sketch the curve $y = \pi(x)$.

(e) If the constant term in our expression for $\pi(x)$ is changed from 312 to 156, how does the answer to (c) change?

(f) Given that the company has a linear cost function and that it costs \$620 to produce four units and \$700 to produce eight units, determine the cost, $C(x)$, of producing x units.

(a) $\pi(x) = -(x^2 - 20x + 100) + 100 + 312$
$= -(x - 10)^2 + 412$

(b) x- intercept : y = 0 $\Rightarrow -(x-10)^2 + 412 = 0$
$\Rightarrow (x-10)^2 = 412 \Rightarrow x-10 = \pm\sqrt{412}$
$\Rightarrow x = 10 \pm \sqrt{412}$
y-intercept : x = 0 $\Rightarrow$ **y = 312 : (0,312)**

(c) Vertex : x = -b/2a = 10 maximises the profit.
Maximum profit , substitute this in the equation:
$\pi(10) = -100 + 200 + 312 = 412$.

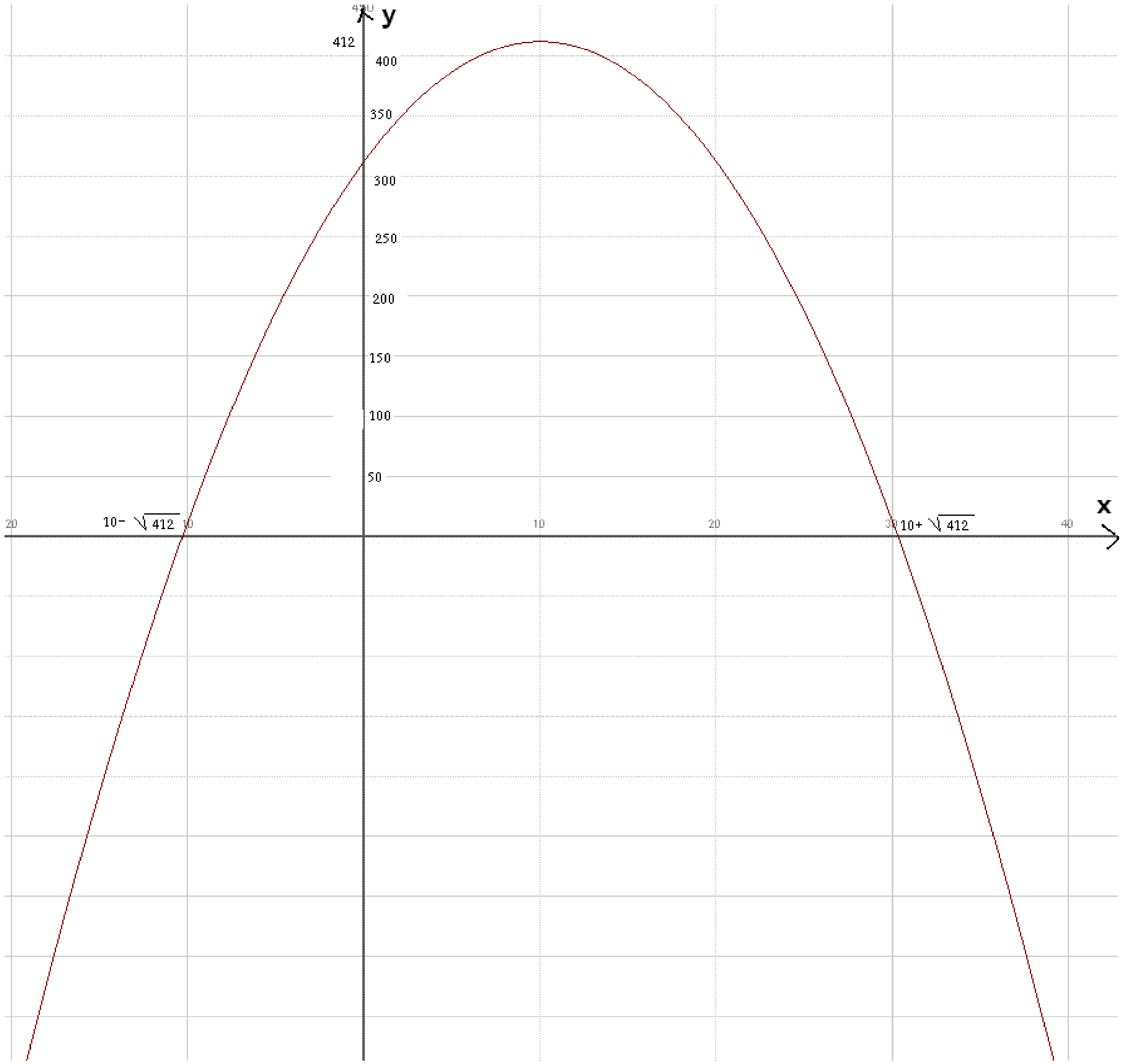
(d) See next page.

(e) The vertex abscissa does not change ,
it is still x = -b/2a = 10
The maximum profit becomes :
$\pi(10) = -100 + 200 + 156 = 256$.

(f) Linear cost function : C = aq + b
For q = 4 , C = 620 $\Rightarrow$ **620 = 4a + b -------(1)**
For q = 8 , C = 700 $\Rightarrow$ **700 = 8a + b -------(2)**
Solving simultaneously , by subtracting (1) from (2):

4a = 80 $\Rightarrow$ **a = 20 ,**
using (1) : b = 620 – 4a = 620 – 80 = 540

$\Rightarrow$ **C = 20q + 540**



END of QUESTIONS