

Unit 05a: Mathematics 1 – (MathA)

Assignment – 1

1. (a) The supply equation for a good is

$$q = 10p^2 + 2p$$

and the demand equation is

$$q = 150 - 6p^2$$

where  $p$  is the price.

Sketch the supply and the demand functions for  $p \geq 0$

Use the graph, or otherwise, to find the positive  $p$  at which the two curves intersect.

(12 Marks)

(b) For which values of  $\alpha \in \mathfrak{R}$  has the equation:

$$x^2 + x + \alpha = 0$$

No solutions, exactly one solution or two solutions?

Determine the solutions in the second and third cases.

(8 Marks)

2. Solve the following equations in the set of real numbers :

a.  $\frac{-5}{7}q + \frac{5}{3}q^2 - \frac{20}{21} = 0$

b. 
$$\begin{cases} -\frac{3}{4}x + 8y - 37 = 0 \\ -35 + 8x + \frac{3}{5}y = 0 \end{cases}$$

c.  $(\ln x)^2 + \ln x^2 - 1 = 0$

d.  $2e^{x^2} + 2x(2x - 3)e^{x^2} = 0$

e.  $\ln x + \ln y = 0$   
 $x + y = 2$

f.  $|7x - 5| - 1 > 10$

g.  $|8x + 1| - 13 < 4$

h.  $|x^2 - 4x + 1| = 4$

i.  $5x - \frac{1}{x} = 4$

j.  $\sqrt{2x - 1} = 2 - 3x$

(20 Marks)

3. The demand for a commodity is given by :  $p(q + 1) = 231$   
and the supply is given by :  $p - q = 11$  .Determine the  
equilibrium price and quantity. **(10 Marks)**

4. A monopolist's average cost function is given by :  $2 + 3q - \frac{5}{q}$

Where  $q$  is the quantity produced, the demand function for the

good is  $q = 10 - \frac{p}{2}$

Determine expressions, in terms of  $q$  , for the revenue and the profit and determine the value of  $q$  that maximizes the profit. Find the maximum profit.

**(10 Marks)**

**5. (20 Marks)**

The inverse supply and demand functions for a market are given by the equations

$$p^S(q) = 2q + 3 \quad \text{and} \quad p^D(q) = -q^2 - 2q + 8,$$

respectively.

- Write  $p^D(q)$  in completed square form and determine the coordinates and nature of the turning point of the curve  $p = p^D(q)$ .
- Determine the  $p$  and  $q$ -intercepts of the curves  $p = p^S(q)$  and  $p = p^D(q)$ .
- Find the points of intersection of the curves  $p = p^S(q)$  and  $p = p^D(q)$ . Hence, deduce the equilibrium price and quantity for this market.
- Sketch both of these curves on the same axes clearly indicating which parts of these curves are economically meaningful.

**6. (20 Marks)**

A company has a profit function given by,

$$\pi(x) = -x^2 + 20x + 312,$$

where  $x$  denotes the quantity produced.

- Write the function  $\pi(x)$  in completed square form.
- Find the  $x$ -intercepts and  $y$ -intercepts of the curve  $y = \pi(x)$ .
- Find the value of  $x$  that gives the maximum profit. What is the maximum profit?
- Use the above information to sketch the curve  $y = \pi(x)$ .
- If the constant term in our expression for  $\pi(x)$  is changed from 312 to 156, how does the answer to (c) change?
- Given that the company has a linear cost function and that it costs \$620 to produce four units and \$700 to produce eight units, determine the cost,  $C(x)$ , of producing  $x$  units.

**END of QUESTIONS**