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UNIVERSITY OF LONDON

279 005b ZA
990 005b ZA

BSc degrees in Economics, Management, Finance and the Social Sciences, the Diploma in Economics and Access Route for Students in the External Programme

Mathematics 2 (half unit)

Tuesday, 13 May 2003 : 2.30pm to 4.30pm

Candidates should answer **EIGHT** of the following **TEN** questions: **SIX** from Section A (60 marks in total) and **TWO** from Section B (20 marks each).

Graph paper is provided. If used, it must be securely fastened inside the answer book.

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UL03/409

SECTION A

Answer all **SIX** questions from this section (60 marks in total)

1 Show that the function

$$f(x, y) = \frac{x^3}{x+y} + xy\cos\left(\frac{x}{y}\right)$$

is homogeneous of degree 2, and verify that

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 2f(x, y).$$

2 Suppose the supply and demand functions for a good are, respectively,

$$q^S(p) = bp - a, \quad q^D(p) = c - dp,$$

where a, b, c, d are positive numbers. Find the equilibrium price and quantity. If a per-unit (excise) tax of T is imposed, find the new equilibrium price and quantity. Find also the amount of tax revenue and determine the value of T that maximises the tax revenue. Show, further, that the tax level that maximises the revenue reduces the sales of the good by exactly half.

3 The demand function for a good is $q = q^D(p)$, and the price elasticity of demand, $\varepsilon = -\frac{p}{q}\frac{dq}{dp}$, satisfies $\varepsilon(p) = 2p^2$. Given that $q^D(1) = 10$, find the demand function $q^D(p)$.

4 Using a matrix method, find all the solutions to the following system of equations:

$$\begin{aligned} 2x_1 - x_2 + x_3 &= 4 \\ x_1 - 3x_2 + 2x_3 &= 4 \\ 3x_1 + x_2 &= 4. \end{aligned}$$

5 Find the eigenvalues of the matrix

$$A = \begin{pmatrix} -5 & -2 \\ 4 & 1 \end{pmatrix}$$

and find an eigenvector corresponding to each eigenvalue. Hence find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$.

Use your result to find the sequences x_t and y_t such that $x_0 = 1$, $y_0 = 2$ and, for $t \geq 1$,

$$\begin{aligned} x_t &= -5x_{t-1} - 2y_{t-1} \\ y_t &= 4x_{t-1} + y_{t-1}. \end{aligned}$$

6 Find the function $f(x)$ satisfying

$$f(0) = 0, \quad f(1) = 0, \quad \frac{d^2 f}{dx^2} - 6\frac{df}{dx} + 9f = 9x - 15.$$

SECTION B

Answer **two** questions from this section (20 marks each)

7 (a) Expand as a power series, in terms up to x^4, the function

$$f(x) = e^{x^2 - x}.$$

(b) Functions $f(t)$ and $g(t)$ are related as follows:

$$\begin{aligned} \frac{df}{dt} &= 5f(t) - g(t) \\ \frac{dg}{dt} &= -2f(t) + 4g(t). \end{aligned}$$

If $f(0) = 2$ and $g(0) = 1$, find $f(t)$ and $g(t)$.

8 (a) Find the inverse of the matrix

$$A = \begin{pmatrix} 2 & -2 & 0 \\ -1 & 3 & 0 \\ 0 & 1 & -1 \end{pmatrix}.$$

(b) The balance $S(t)$ of a bank account at time t is subject to continuous compounding and a net outflow. The balance varies with time according to the equation

$$\frac{dS}{dt} = \frac{S(t)}{20} - t - 2.$$

The balance at time 0 is P. By solving this differential equation, find $S(t)$.

Suppose that $P = 300$. Show that the balance $S(t)$ initially increases, to a maximum value, and thereafter decreases.

9 (a) Suppose the demand function for a commodity is given by

$$q = \frac{p}{(p^2 + 18)^3}.$$

Find the elasticity of demand, in terms of p. Determine the values of p for which the demand is inelastic.

(b) The sequence y_t has the property that, for $t \geq 1$,

$$y_t = cy_{t+1} + (1 - c)y_{t-1},$$

where c is a fixed number between 0 and 1/2. Show that

$$y_t = A + B\left(\frac{1-c}{c}\right)^t,$$

for some numbers A and B.

10 (a) Find the values of x, y, z that minimise the function

$$u(x, y, z) = x^4 + y^4 + z^4,$$

subject to the constraint $x + 8y + 27z = 10$.

(b) Suppose that the price $p(t)$ of a good varies continuously with time, and that the quantities demanded and supplied are given, respectively, by

$$q^D(p) = 1 - p, \quad q^S(p) = p.$$

Price adjusts according to the rule

$$\frac{dp}{dt} = \left(q^D(p) - q^S(p)\right)^5.$$

Find an expression for $p(t)$, given that when $t = 0$ the price is 1/4. How does the price behave as t tends to infinity?

END OF PAPER