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UNIVERSITY OF LONDON

279 005b ZA
990 005b ZA
996 D05b ZA

BSc degrees in Economics, Management, Finance and the Social Sciences, the Diploma in Economics and Access Route for Students in the External Programme

Mathematics 2 (half unit)

Tuesday, 11 May 2004 : 2.30pm to 4.30pm

Candidates should answer **EIGHT** of the following **TEN** questions: **SIX** from Section A (60 marks in total) and **TWO** from Section B (20 marks each).

Graph paper is provided. If used, it must be securely fastened inside the answer book.

Calculators may **not** be used for this paper.

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UL04/409

SECTION A

Answer all **SIX** questions from this section (60 marks in total)

1 The demand equation for a good is $p(q+3) = 15$, and the supply equation is $q = p - 1$. Find the equilibrium price and quantity. Show that the consumer surplus is

$$15 \ln\left(\frac{5}{3}\right) - 6.$$

Find also the producer surplus.

2 Show that the function

$$f(x, y) = x \sin\left(\frac{x}{y}\right) + xe^{-y/x}$$

(defined for positive x and y) is homogeneous and verify that Euler's equation holds.

3 Find the function $f(x)$ such that $f(0) = 2$, $f'(0) = 11$ and

$$\frac{d^2 f}{dx^2} = -\frac{df}{dx} + 12f - 12x - 11.$$

4 Using matrix methods, throughout, show that there is just one value of k for which the following system of linear equations has more than one solution, and determine all the solutions when k takes this value. What is the solution for other values of k?

$$\begin{aligned} 2x + y + z &= 4 \\ 6x - y - z &= 4 \\ -4x + ky + 6z &= 8. \end{aligned}$$

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5 Find the eigenvalues of the matrix

$$A = \begin{pmatrix} 5 & -1 \\ -3 & 7 \end{pmatrix}$$

and find an eigenvector corresponding to each eigenvalue. Hence find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$.

Use your result to find the functions $f(t)$ and $g(t)$ such that $f(0) = 1$, $g(0) = -1$ and

$$\begin{aligned} \frac{df}{dt} &= 5f(t) - g(t) \\ \frac{dg}{dt} &= -3f(t) + 7g(t). \end{aligned}$$

6 The function $y(x)$ is such that $y(x) > 0$ for all x, $y(0) = 1$ and

$$\frac{dy}{dx} = \frac{x}{y\sqrt{1+x^2}}.$$

Find an expression for $y(x)$ in terms of x.

SECTION B

Answer **two** questions from this section (20 marks each)

7 (a) Expand as a power series, in terms up to x^4, the function

$$f(x) = \frac{\ln(1-x)}{(1-x)}.$$

(b) The demand function for a good is $q = q^D(p)$, and the price elasticity of demand, $\varepsilon = -\frac{p}{q}\frac{dq}{dp}$, satisfies

$$\varepsilon(p) = \frac{p^2}{p^2 + 3p + 2}.$$

Given that $q^D(1) = 4$, find the demand function $q^D(p)$.

8 (a) Find the inverse of the matrix

$$A = \begin{pmatrix} 2 & -1 & -2 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}.$$

(b) Suppose the supply and demand functions for a good are, respectively,

$$q^S(p) = p - 5, \quad q^D(p) = 7 - p.$$

Determine the equilibrium price and quantity.

Suppose a percentage tax of $R\%$ (that is, $R\%$ of the price) is imposed. Find the new equilibrium price and quantity.

Find the new equilibrium price and quantity if, instead of a percentage tax, an excise (per-unit) tax of T is imposed.

Suppose that the equilibrium price in the presence of an excise tax of T per unit is the same as the equilibrium price in the presence of a percentage tax of $R\%$. Find a formula for T in terms of R.

9 (a) Suppose that $C > 0$. Use the Lagrange multiplier method to find the maximum value of

$$f(x, y, z) = \frac{x}{(1+x)} \frac{y}{(1+y)} \frac{z}{(1+z)}$$

among all positive x, y and z satisfying $x + y + z = C$.

(b) A sequence of numbers x_t is defined as follows: $x_0 = 1$, $x_1 = 1$ and, for $t \geq 2$,

$$x_t = 6x_{t-2} - x_{t-1}.$$

Find an expression, in terms of t, for x_t.

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10 (a) Find the sequences q_t and p_t such that $q_0 = 2$ and, for all $t \geq 1$,

$$\begin{aligned} q_t &= 3 - 2p_t \\ q_t &= 2 + 0.5p_{t-1}. \end{aligned}$$

How do p_t and q_t behave as t tends to infinity?

(b) Find the function $y(x)$ satisfying $y(0) = 1$ and

$$\frac{dy}{dx} + \frac{2xy}{x^2 + 1} = x^3\sqrt{x^2 + 1}.$$

END OF PAPER