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Integration IV Handout #18

Integration Techniques

Integration of functions that require more than one integration method:

1. Substitution and partial fractions:

Example1

$$\int \frac{dx}{\cos^2 x(\tan^2 x + 6\tan x + 8)} \text{ , u = tanx , du = dx/cos}^2\text{x}$$

$$= \int \frac{du}{u^2 + 6u + 8} \text{ , } \frac{1}{u^2 + 6u + 8} = \frac{a}{u+2} + \frac{b}{u+4}$$

1 = a(u+4) + b(u+2) $\Rightarrow$ a = ½ , b = - ½

$$\int \frac{du}{u^2 + 6u + 8} = ½ \ln |u+2| - ½ \ln|u+4| + C = ½ \ln\left|\frac{u+2}{u+4}\right| + C$$

$$= ½ \ln\left|\frac{\tan x + 2}{\tan x + 4}\right| + C$$

Example2

$$\int \frac{dx}{x^{1/2} - x^{3/2}} = \int \frac{dx}{x^{1/2}(1-x)} = \int \frac{dx}{\sqrt{x}(1-\sqrt{x})(1+\sqrt{x})}$$

$$u = \sqrt{x} \Rightarrow du = \frac{dx}{2\sqrt{x}}$$

$$= 2 \int \frac{du}{(1-u)(1+u)} \text{ , } \frac{1}{(1-u)(1+u)} = \frac{a}{1-u} + \frac{b}{1+u}$$

1 = a(1- u) + b(1+u) $\Rightarrow$ a = ½ , b = ½

$$2\int \frac{du}{(1-u)(1+u)} = 2 [- ½ \ln|1-u| + ½ \ln|1+u|] + C$$

$$2 [½ \ln\left|\frac{1+u}{1-u}\right|] + C = \ln\left|\frac{1+u}{1-u}\right| + C = \ln\left|\frac{1+\sqrt{x}}{1-\sqrt{x}}\right| + C$$

Example3

$\int \frac{dx}{e^x - e^{-x}} = \int \frac{dx}{e^x - \frac{1}{e^x}} = \int \frac{e^x dx}{e^{2x} - 1}$, u = e^x , du = e^x dx

= $\int \frac{du}{u^2 - 1}$, $\frac{1}{(u-1)(u+1)} = \frac{a}{u-1} + \frac{b}{u+1}$

1 = a(u - 1) + b(u+1) $\Rightarrow$ a = - ½ , b = ½

$\int \frac{du}{u^2 - 1}$ = -½ ln|u-1| + ½ ln|u+1| + C= ½ $\ln\left|\frac{u+1}{u-1}\right| + C$ =½ $\ln\left|\frac{e^x+1}{e^x-1}\right| + C$

2. Substitution and by parts:

$\int x^5 e^{x^3} dx$, **u = x^3 + 1 $\Rightarrow$ du = $3x^2$ dx $\Rightarrow$ dx =** $\frac{du}{3x^2}$

$\int x^5 e^{x^3} dx$ = $\int x^5 e^u \frac{du}{3x^2}$ = $\frac{1}{3}\int x^3 e^u du$ **But u = x^3 ,**

$\frac{1}{3}\int u e^u du$ **= ue^u – e^u + C (By Parts) =(1/3)** (x^3 e^{x^3} - e^{x^3}) + C

3. Getting the original integral when using by parts method:

$\int e^x \sin x dx$,by parts

Let u = sinx $\Rightarrow$ du = cosx dx

Let dv = e^x dx $\Rightarrow$ v = e^x

$\int e^x \sin x dx$ = e^x sinx - $\int e^x \cos x dx$

$\int e^x \cos x dx$ by parts again :

u = cosx $\Rightarrow$ du = - sinx dx

dv = e^x dx $\Rightarrow$ v = e^x

$\int e^x \cos x dx$ = e^x cosx - $\int e^x(-\sin x)dx$ = e^x cosx + $\int e^x \sin x dx$

$\int e^x \sin x dx$ = e^x sinx - e^x cosx - $\int e^x \sin x dx$

We reached the original integral,it is useless to do it by parts again,instead we solve it as an equation in the original integral:

$\int e^x \sin x dx$ = e^x sinx - e^x cosx - $\int e^x \sin x dx$; Let **I** = $\int e^x \sin x dx$

I = e^x sinx - e^x cosx – **I** $\Rightarrow$ **2I** = e^x sinx - e^x cosx

$\Rightarrow$ **I** = (1/2)(e^x sinx - e^x cosx) = ½ e^x (sinx – cosx) + C

Exercise : $\int \cos(\ln x)dx$