Sharjah Institute of Technology

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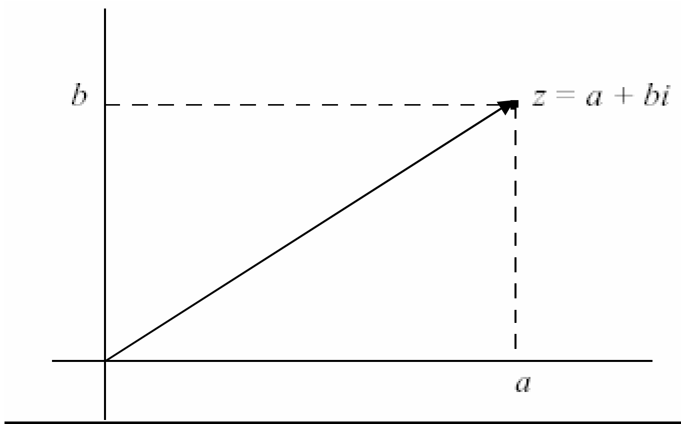
Handout #6

Imaginary Numbers

Algebraic Form : z = a + i b ; a = real part ; b = imaginary part ; $i^2 = -1$

Argand diagram :



Conjugate: z = a + ib ,the conjugate $\bar{z}$ = a – ib ;

$z\bar{z} = a^2 + b^2$, for e.g.

$$\frac{2+i}{1+i} = \frac{2+i}{1+i}\frac{1-i}{1-i} = \frac{2-2i+i+1}{1+1} = \frac{3-i}{2} = \frac{3}{2} - \frac{1}{2}i.$$

Magnitude(Modulus): | z | = $\sqrt{z\bar{z}}$ **=** $\sqrt{a^2+b^2}$

Polar (trigonometric) Form :

z = r(cosθ + isinθ) = re$^{i\theta}$; r = |z| = $\sqrt{x^2+y^2}$;

tanθ = y/x ; θ =Argz (argument of z)[Recall Euler's : $e^{i\theta}$ = cosθ + isinθ]

e.g. z=1- i

r = $\sqrt{2}$ **;**

tanθ = -1;θ = -π/4;

z= $\sqrt{2}\,e^{-\pi/4i}$ = $\sqrt{2}$ [cos(-π/4)+isin(-π/4)]

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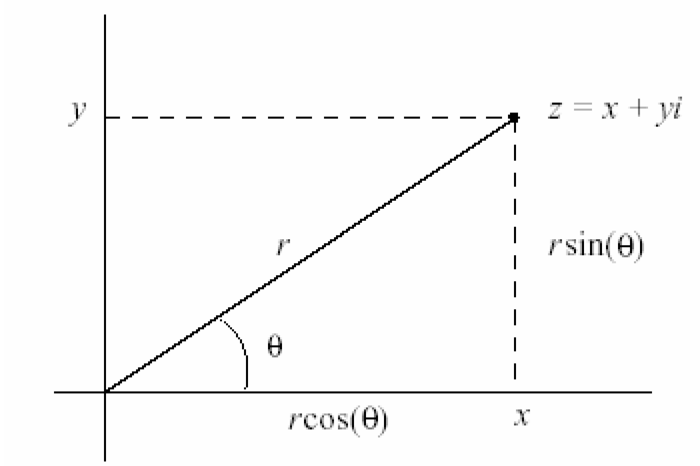

Demiovre's

$(\cos\theta + i\sin\theta)^n = e^{in\theta} = \cos(n\theta) + i\sin(n\theta)$

$z = r(\cos\theta + i\sin\theta)$; $z^n = r^n(\cos\theta + i\sin\theta)^n = r^n e^{in\theta} = r^n[\cos(n\theta) + i\sin(n\theta)]$

e.g. $(-1 - i\sqrt{3})^6$:

$r = 2$; $\theta = \pi/3$;

$[2(\cos(\pi/3)+i\sin(\pi/3))]^6 = 2^6[\cos 6(\pi/3)+i\sin 6(\pi/3)] = 64[\cos 2\pi+i\sin 2\pi]$

$= 64[1 + i(0)] = 64$

Example : Write Ln $(\frac{1}{2}-\frac{\sqrt{3}}{2}\boldsymbol{i})$ in the form a + ib

Use Euler's identity : $r\, e^{i\theta} = r(\cos\theta + i\sin\theta)$

Now you need to convert $\frac{1}{2}\left(1 - i\sqrt{3}\right) = \frac{1}{2} - \frac{\sqrt{3}}{2}\boldsymbol{i}$ to trigonometric form.

z = a + ib is converted to z = r(cosθ + isinθ) where

r = $\sqrt{a^2 + b^2}$ and θ = $\tan^{-1}\frac{b}{a}$

For $\frac{1}{2} - \frac{\sqrt{3}}{2}\boldsymbol{i}$: r = $\sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = 1$; θ = $\tan^{-1}(-\sqrt{3})$ = -60°

(your calculator mode in degrees:Now convert this to radians : $-60 \times \frac{\pi}{180} = \frac{-\pi}{3}$ rd; you may **add** 2π to get rid of the –ve sign: $\theta = \frac{5\pi}{3}$)

Hence : $\frac{1}{2} - \frac{\sqrt{3}}{2}\boldsymbol{i} = e^{i\frac{5\pi}{3}} \Rightarrow \text{Ln}\,(\frac{1}{2} - \frac{\sqrt{3}}{2}\boldsymbol{i}) = \text{Ln}\, e^{i\frac{5\pi}{3}} = i\frac{5\pi}{3}$